ADVANCED NUMERICAL METHODS

BACHELOR'S DEGREE IN INDUSTRIAL TECHNOLOGY ENGINEERING

JUNE 19, 2017

TIME: 3 hours **30 points total**

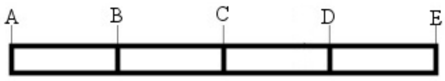
N.B. All exercises in which you need to use a calculator should be solved with rounding to six or more significant digits.

1.- a) Obtain the Taylor polynomial p_3 of degree ≤ 3 of $f(x) = \cos(2x)$ at $x_0 = 0$ *from a table of differences*, specifying the interpolation data used. (2.5 points)

b) Write the general error term of osculating polynomials and particularize it for the polynomial p_3 of the previous section. (1.5 points)

c) One now looks for a polynomial p_5 of degree ≤ 5 that, besides the above conditions at $x_0 = 0$, also satisfies $p_5(-1) = p_5(1) = -1$. Complete the table of section a) and find p_5 and its truncation error. Does p_5 interpolate $f(x) = \cos(2x)$? (2 points)

2.- A body moving along a 10 m-long underground pipe emits a signal whose intensity could be measured at 5 equally-spaced points according to the scheme below; the intensities obtained are the ones shown in the table.



A ($x=0$)	B	C	D	E ($x=10$)
1.61534	2.18174	2.25203	1.98124	1.56423

There seems to be an intermediate point with maximum signal. Estimate it by optimally *evaluating* an interpolation polynomial that uses these data: (3.5 points)

x_i	1.25	3.75	6.25	8.75
$f'(x_i)$	0.56640	0.07029	−0.27079	−0.41701

3.- a) *Derive* Simpson's simple quadrature rule by integrating an interpolation polynomial, and obtain its truncation error term. (3.5 points)

b) *Derive* the compound Simpson rule and its truncation error. (2.5 points)

c) Estimate $\int_0^{0.8} f(x)dx$ using the compound Simpson rule and these data: (1 point)

x_i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(x_i)$	0	2.1220	3.0244	3.2568	3.1399	2.8579	2.5140	2.1639	1.8358

d) Find a number of subintervals for the compound Simpson rule guaranteeing that the integral $\int_{0.5}^{1} e^{x^2} dx$ is calculated with error less than 10^{-8}. (1.5 points)

4.- Given the differential problem $\begin{cases} y'' - 2y = \cos(t) - e^t \\ y(0) = -1, \quad y'(0) = 2 \end{cases}$

take *two* steps of the fourth-order Runge-Kutta method to estimate its solution at $t = 0.2$.

(5.5 points)

5.- What interval of the form $[10, b]$ will be adequate to solve, without instability problems, the initial-value problem $\begin{cases} y' = t^2 - ty \\ y(10) = 3 \end{cases}$ $(t \geq 10)$ using the fourth-order Runge-Kutta method with step size $h = 0.2$? (1.5 points)

N.B. The absolute stability interval of the RK4 method is $(-2.78, 0)$.

6.- Justify which of the following expressions can be truncation error terms of an interpolatory numerical differentiation formula to approximate $f'(z)$. For the affirmative cases, describe explicitly some valid positions of the nodes:

$$E_1 = \frac{f^{3)}(\xi)}{3!}\Pi(z)\,; \quad E_2 = \frac{f^{4)}(\xi)}{4!}\Pi(z)\,; \quad E_3 = \frac{f^{3)}(\xi)}{3!}\Pi'(z)$$

$$E_4 = \frac{f^{3)}(\xi)}{3!}\Pi(z) + \frac{f^{4)}(\eta)}{4!}\Pi'(z)\,; \quad E_5 = \frac{f^{3)}(\xi)}{3!}\Pi'(z) + \frac{f^{4)}(\eta)}{4!}\Pi(z)$$

for some ξ, η in an interval containing the nodes. (2.5 points)

7.- Find an interpolatory numerical differentiation formula (without its truncation error term) to estimate $f''(z)$ in terms of the values of f at $z, \quad z+2h, \quad z+3h.$ (2.5 points)