ADVANCED NUMERICAL METHODS
DEGREE IN INDUSTRIAL TECHNOLOGY

MAY 27, 2016

TIME: 3 hours

N.B. **All the exercises requiring the use of a calculator will be solved rounding to 6 significant digits.**

1.- Is a polynomial uniquely determined from the following data?

$$p(-1) = f(-1), \quad p'(-1) = f'(-1), \quad p'(1) = f'(1), \quad p''(2) = f''(2)\ ?$$

Reason out the answer. (1.5p)

2.-A) Calculate the interpolation polynomial verifying the data in the following table:

x_i	$f(x_i)$	$f'(x_i)$	$f''(x_i)$
-1	-1	4	-6
1	3	4	

Evaluate it **as optimally as possible** at point 0.5. (3p)

B) Knowing that $f''(1) = 8$, estimate the error made in the previous section. (1.5p)

3.- Which of the following matrices could be the coefficient matrix T of the system to be solved to calculate a natural cubic spline with 5 nodes? Reason out the answer.

$$\begin{pmatrix} 0.4 & 0.2 & 0 \\ 0.2 & 0.5 & 0.3 \\ 0 & 0.3 & 0.6 \end{pmatrix} \qquad \begin{pmatrix} 0.6 & 0.2 & 0 \\ 0.2 & 0.8 & 0.2 \\ 0 & 0.2 & 0.6 \end{pmatrix} \qquad \begin{pmatrix} 0.2 & 0.1 & 0 & 0 & 0 \\ 0.1 & 0.6 & 0.2 & 0 & 0 \\ 0 & 0.2 & 0.6 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0.6 & 0.2 \\ 0 & 0 & 0 & 0.2 & 0.4 \end{pmatrix}$$

(1 p)

4.- Calculate the **exact value** of the following integral

$$\int_1^2 \left[(x-1)^2 + \frac{x^3}{\sqrt{(x-1)(2-x)}} \right] dx$$

using numerical quadrature rules. (4p)

5.- **Study** the stability of quadrature rules with positive coefficients. (2p)

6.- Given the differential problem

$$\begin{cases} y'' + t\,y' + y = 0 \\ y(0) = 1,\ \ y'(0) = 2 \end{cases}$$

A) Take one step of the 4th-order Runge-Kutta method to estimate $y(0.2)$, $y'(0.2)$. (4p)

B) Knowing that

t_i	$y(t_i)$	$y'(t_i)$
0.2	1.37489	1.72502
0.4	1.68176	1.32730
0.6	1.90110	0.859340

estimate $y(0.8)$ and $y'(0.8)$ using Adams's predictor-corrector method with precision 0.1%

$$y_{n+1} = y_n + \frac{h}{24}[55 \cdot f_n - 59 \cdot f_{n-1} + 37 \cdot f_{n-2} - 9 \cdot f_{n-3}] \qquad n = 3, 4, ..., N-1$$

$$y_{n+1} = y_n + \frac{h}{24}[9 \cdot f_{n+1} + 19 \cdot f_n - 5 \cdot f_{n-1} + f_{n-2}] \qquad n = 2, 3, ..., N-1$$ (3p)

7.-A) **Define** the concepts of weak stability and absolute stability region. (1p)

B) Describe the process that you would follow to obtain the absolute stability region of a linear multistep method. (1p)

8.-A) Using base functions, obtain a formula to estimate $f''(z)$ from the values $f(z-2h)$, $f(z-h)$ and $f(z)$. (2p)

B) **Derive** the truncation error made by any interpolatory differentiation formula for the estimation of the second derivative of a function at a point. (2p)

C) From the previous general expression, what is the error term of the formula of section A)? (1p)