ADVANCED NUMERICAL METHODS

DEGREE IN INDUSTRIAL TECHNOLOGY

MAY 12, 2015

PART 1

1.- a) Given the following table of data:

x_i	1.1	1.2	1.3	1.4	1.5	1.6
$f(x_i)$	0.14927	0.14358	0.13065	0.10998	0.08120	0.04406

approximate $f(1.37)$ by means of an interpolation polynomial of degree 3, using the most appropriate nodes and evaluating the polynomial optimally. Estimate the error made in the approximation. Operate with rounding to 5 decimal digits. (2.5p)

b) Using the nodes 1.2, 1.3, 1.4 and 1.5, estimate the value of x for which $f(x) = 0.12$. Operate with rounding to 5 significant digits. (2p)

2.- From this expression of the truncation error of the interpolation polynomial:

$$e(x) = f[x_0, \ldots, x_n, x]\Pi(x) \quad \text{with} \quad \Pi(x) = (x - x_0)(x - x_1)\cdots(x - x_n)$$

prove that when the nodes are uniformly spaced, and knowing $f(x_{n+1})$, that error can be approximated by the expression

$$e(x) \approx \frac{\Delta^{n+1} f(x_0)}{(n+1)!} t(t-1)(t-2)\cdots(t-n) \quad \text{with} \quad x = x_0 + th \quad \text{and} \quad h = x_{i+1} - x_i$$

(0.75p)

3.- Define the Chebyshev polynomials of the 1st kind, $T_n(t)$. State and prove the recurrence relation they verify and obtain their roots. Particularize to $T_4(t)$. (1.5p)

4.- a) Obtain a quadrature rule of interpolatory kind the form

$$\int_0^h f(x)dx \approx hA_0 f(h/2) + h^2 \left[B_0 f'(0) + B_1 f'(h)\right]$$

and obtain the expression of its error term. (2p)

b) Derive the expression resulting from the composition of that formula N times in the interval $[a,b]$. (2p)

c) Compare this formula with the compound Newton-Cotes rules of the same polynomial degree of exactitude. (1p)

d) Apply the compound formula with $N = 3$ to the approximation of $\int_0^{\pi} \cos(x)dx$, and find a bound of the error made. (0.5p)

TIME: 1 hour and 45 minutes

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PART 2

1.- Solve the differential equation

$$y''' - y'' + 2y = \log t$$

subject to $y(1) = -1$, $y'(1) = 0$, $y''(1) = 1$, using the ***Enhanced Euler Method*** to estimate the solution and its derivatives at $t = 1.1$ and $t = 1.2$. Operate with rounding to 6 significant digits. (3.5p)

2.- Write the general expression of a linear multistep method. Comment on when it is explicit or implicit. Write the associated characteristic polynomials. Enunciate the conditions it must verify to be convergent. (0.75p)

3.- Obtain a formula to estimate $f''(z)$ from $f(z-2h)$, $f(z)$ and $f(z+h)$:

a) From the interpolation polynomial $p_2(x)$. (1.25p)

b) Using Taylor series expansions. (1.25p)

c) Obtain the truncation error from the one made in the interpolation. (1p)

TIME: 1 hour and 15 minutes