ADVANCED NUMERICAL METHODS

DEGREE IN INDUSTRIAL TECHNOLOGY ENGINEERING

MAY 22, 2014

PART 1

1.- One wants to model the electricity consumption of a household along the year from the following available information:

- The maximum monthly consumption, which takes place in February, is 112 kW;
- In August the consumption is 49 kW, and it is the minimum one in the year;
- For the month of May the counter reads 76 kW.

Giving the results rounded to 6 decimal digits:

A) Construct the interpolation polynomial of maximum degree with the data provided, and use it *optimally* to estimate the consumption in April. **(2p)**

B) Estimate the error made before if the March consumption was 104 kW. **(0.5p)**

2.- Reason out and find the values of α, β and γ so that the following function is a cubic spline:

$$s(x) = \begin{cases} \alpha x^3 + \gamma x & \text{if} \quad 0 \le x \le 1 \\ -\alpha x^3 + \beta x^2 - 5\alpha x + 1 & \text{if} \quad 1 \le x \le 2 \end{cases}$$

With the values obtained, is it a natural cubic spline, or one with boundary conditions? **(2p)**

3.- Describe the *Runge effect* and its characteristics depending on the nodes used. **(1p)**

4.- Giving the results rounded to 6 decimal digits, use Gaussian quadrature to approximate the following integral with precision (termination criterion) 0.01%:

$$I = \int_2^3 \frac{L(x)}{\sqrt{(x-2)(3-x)}}\,dx$$ **(2.5p)**

5.- Given the following formula of numerical integration:

$$\int_a^b f(x)dx = A_0 f(x_0) + A_1 f(x_1) + A_2 f(x_2) + \frac{14h^5}{45} f^{4)}(\xi) \quad \text{for some } \xi \in (a,b)$$

A) Without calculating the coefficients, what kind of formula do you think it is? *Reason out your answer* according to its properties. **(1.25p)**

B) Obtain the coefficients of the formula. **(0.75p)**

TIME: 1 hour and 30 minutes

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PART 2

1.- The trajectory of a particle on the plane is described by the following system of differential equations:

$$\begin{cases} x'(t) = t \cdot x(t) + y(t) \\ y'(t) = x(t) + t \end{cases}$$

Knowing that at the initial instant the particle is located at point (1,–1) of the plane, use the *Modified Euler method* to estimate its position at instants 0.1 s and 0.2 s (use step size $h = 0.1$ seconds). Give the results rounded to 6 significant digits. **(3p)**

2.- Study the convergence and the order of convergence of the following multistep linear method for the resolution of initial value problems:

$$y_{n+1} - y_{n-1} = \frac{h}{3}\left(7f_n - 2f_{n-1} + f_{n-2}\right)$$

Reason out the characteristics of the method: Is it explicit or implicit? Adams-Bashforth, Adams-Moulton or some other type? What's the number of steps? **(3.5p)**

3.- Using Taylor series expansions, obtain an expression of the highest possible order to estimate $f'(z)$ from the information provided by $f(z-h)$, $f(z)$ and $f(z+2h)$. Calculate as well the truncation error from the Taylor expansions used. **(3.5p)**

TIME: 1 hour and 30 minutes