ADIB. EULER METODOAZ $\longrightarrow \begin{cases} y(0,2)\ ? \\ z(0,2)\ ? \end{cases}$ $\boxed{h=0,1}$ IZANIK

y(x) ETA z(x) IZANIK

$$\begin{cases} y' = yz + x \\ z' = x \cdot z + y \end{cases} \qquad \begin{cases} u_1 = y \\ u_2 = z \end{cases} \qquad \underset{\sim}{u}(x) = \begin{pmatrix} u_1(x) \\ u_2(x) \end{pmatrix} = \begin{pmatrix} y(x) \\ z(x) \end{pmatrix}$$

$$\begin{cases} y(0) = 1 \\ z(0) = -1 \end{cases}$$ HASTAPEN BALDINTZAK

$$\underset{\sim}{x}' = f(t, x)$$

$$\underset{\sim}{u}' = \begin{pmatrix} y' \\ z' \end{pmatrix} = \begin{pmatrix} yz + x \\ xz + y \end{pmatrix} = \underbrace{\begin{pmatrix} u_1 u_2 + t \\ t u_2 + u_1 \end{pmatrix}}_{f(t,y)} \qquad f\left(t, \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\right) = \begin{pmatrix} u_1 u_2 + t \\ t u_2 + u_1 \end{pmatrix}$$

$$\underset{\sim}{u}_0 = \begin{pmatrix} y(0) \\ z(0) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Hastapen Baldintza

$$\underset{t_0}{0} \ \underbrace{\qquad}_{h}\ \underset{t_1}{0,1}\ \underbrace{\qquad}_{h}\ \underset{t_2}{0,2}$$

PAUSUAK

$\begin{cases} y(0,1) \\ z(0,1) \end{cases}$

- $\underset{\sim}{u}_1 = \underset{\sim}{u}_0 + h\, f(t_0, \underset{\sim}{u}_0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + 0,1\, f\left(0, \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) =$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} + 0,1 \cdot \begin{pmatrix} 1(-1) + 0 \\ 0(-1) + 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + 0,1 \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 - 0,1 \\ -1 + 0,1 \end{pmatrix} = \begin{pmatrix} 0,9 \\ -0,9 \end{pmatrix} \begin{matrix} \approx y(0,1) \\ \approx z(0,1) \end{matrix}$$

$\begin{cases} y(0,2) \\ z(0,2) \end{cases}$

- $\underset{\sim}{u}_2 = \underset{\sim}{u}_1 + h\, f(t_1, \underset{\sim}{u}_1) = \begin{pmatrix} 0,9 \\ -0,9 \end{pmatrix} + 0,1\, f\left(0,1, \begin{pmatrix} 0,9 \\ -0,9 \end{pmatrix}\right) =$

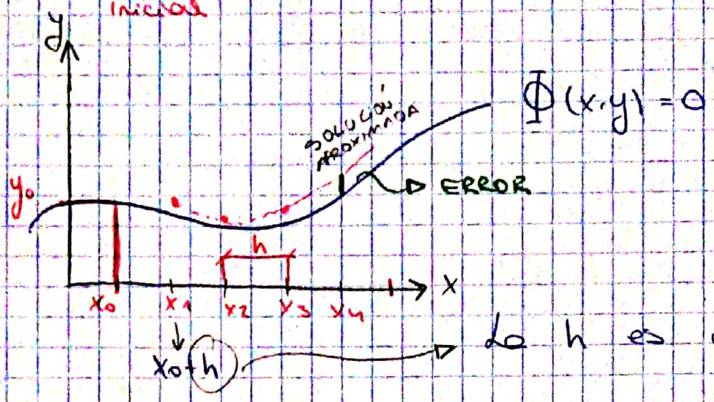
$$= \begin{pmatrix} 0,9 \\ -0,9 \end{pmatrix} + 0,1 \begin{pmatrix} 0,9(-0,9) + 0,1 \\ 0,1(-0,9) + 0,9 \end{pmatrix} = \begin{pmatrix} 0,829 \\ -0,819 \end{pmatrix} \boxed{\begin{matrix} \approx y(0,2) \\ \approx z(0,2) \end{matrix}}$$

METODOS NUMÉRICOS PARA $\begin{cases} \text{EDA} \\ \text{SISTEMAS DE EDA} \end{cases}$

EDA $\quad y' = f(x,y) \xrightarrow{\text{SOLUCIÓN}} \Phi(x,y) = K$ SOL. GENERAL

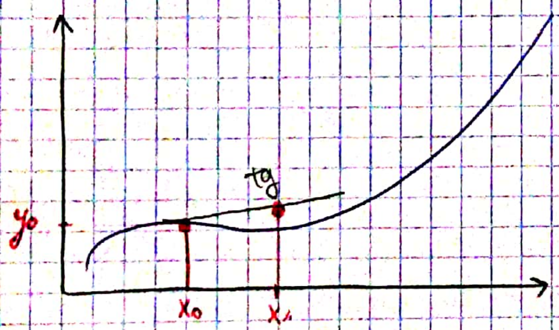
C.I: $y(x_0) = y_0$ ——— $\Phi(x,y) = 0$ SOL. PARTICULAR

Condición inicial



La h es constante

• EULER



El error es cada vez más grande

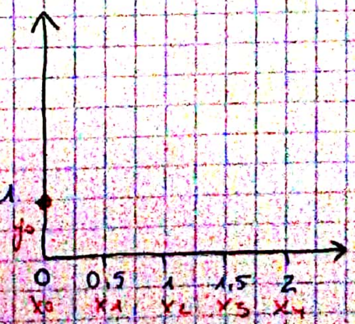
$$y_{i+1} = y_i + h \cdot f(x_i, y_i)$$

TARTEEN LUZERA $= h = \dfrac{b-a}{n} = \dfrac{2-0}{n} = 0{,}5 ; \; \boxed{n=4}$ 4 TARTE

■ EJEMPLO

$y' = f(x,y)$

Resolver la ecuación diferencial ~~con~~ $y' = x \cdot y$ con la condición inicial $y(0) = 1$ en el intervalo $[0,2]$ y $h = 0.5$



$i=0$ • $y_1 = y_0 + h \cdot f(x_0, y_0) = 1 + 0{,}5 \cdot 0 \cdot 1 = 1$

$i=1$ • $y_2 = y_1 + h \cdot f(x_1, y_1) = 1 + 0{,}5 \cdot 0{,}5 \cdot 1 = 1{,}25$

$i=2$ • $y_3 = y_2 + h \cdot f(x_2, y_2) = 1{,}25 + \overset{0,625}{0{,}5 \cdot 1 \cdot 1{,}25} = 1{,}875$

$i=3$ • $y_4 = y_3 + h \cdot f(x_3, y_3) = 1{,}875 + 0{,}5 \cdot 1{,}5 \cdot 1{,}875 = 3{,}28125$

- SOL. EXACTA

$\hookrightarrow \dfrac{dy}{dx} = x\cdot y \; ; \; \dfrac{dy}{y} = x\,dx = \ln|y| = \dfrac{x^2}{2} + C$

y bakandu

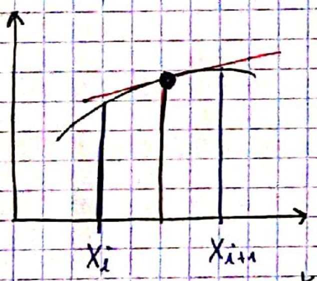
Solución general: $y = Ke^{-\frac{x^2}{2}}$

CONDICIÓN GENERAL $\left\{ \begin{array}{l} x=0 \\ y=1 \end{array} \right. \rightsquigarrow \boxed{K=1}$

x_0	0	1	1	0
x_1	0,5	1	1,13315	0,13315
x_2	1	1,25	1,64872	0,398721
x_3	1,5	1,875	~~1,~~ ___	___
x_4	2	3,28125	___	___
		SOLUCIÓN APROXIMADA	SOLUCIÓN EXACTA	ERROR

ERROR → SOL. APROX $-$ SOL. EXACT

- HEUN (EULER MEJORADO)



EDO

$$y_{i+1} = y_i + \frac{h}{2}\Big[\overbrace{f(x_i, y_i)}^{k_1} + \overbrace{f\big(x_i+h,\; y_i + h\underbrace{f(x_i, y_i)}_{k_1}\big)}^{k_2}\Big]$$

$$y_{i+1} = y_i + \frac{k_1+k_2}{2}\cdot h \qquad \left\{ \begin{array}{l} k_1 = f(x_i, y_i) \\ k_2 = f(x_i+h,\; y_i + k_1\cdot h) \end{array} \right.$$

EJEMPLO

$y' = x\cdot y + \dfrac{x}{y} \qquad y(1) = 2 \qquad h = 0,1 \qquad x\in[1,2]$

Metodo Heun $\quad x = 1,6 \quad$ sol. APROX $\;$ 4,76655

7,962 $\quad x = 1,7 \quad$?

$$y_7 = y_6 + \frac{h}{2}\Big[\underbrace{\Big(1,6\cdot 4,76655 + \frac{1,6}{4,76655}\Big)}_{k_1} + \underbrace{(1,6+h)\cdot(4,76655 + h\cdot 7,762) + \frac{1,6+h}{4,76655+h\cdot 7,762}}_{k_2}\Big]$$

$y_7 = 5,65277$

- RUNGE - KUTTA (RK4)

$$y_{i+1} = y_i + h\,\frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$

$$\begin{cases} k_1 = f(x_i, y_i) \\ k_2 = f\left(x_i + \frac{h}{2},\ y_i + \frac{h}{2}k_1\right) \\ k_3 = f\left(x_i + \frac{h}{2},\ y_i + \frac{h}{2}k_2\right) \\ k_4 = f(x_i + h,\ y_i + hk_3) \end{cases}$$

■ EJEMPLO

$y' = \frac{1}{2}(1+x)\cdot y^2$

$y(0) = 1$

$[0, 0.5]$, $h = 0.1$

A) EULER

B) HEUN

C) RK4

⊗ TABLA con las aproximaciones de cada método

2020-01-03

SISTEMAS DE ECUACIONES DIFERENCIALES DE PRIMER ORDEN

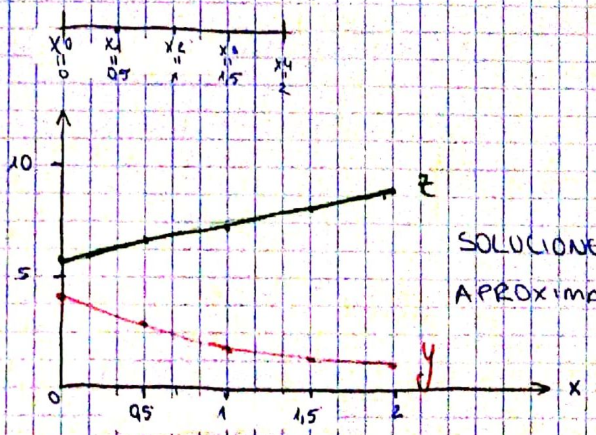
■ $\begin{pmatrix} y' \\ z' \end{pmatrix} = \boxed{\begin{pmatrix} -0.5y \\ 4-0.3z-0.1y \end{pmatrix}}$ $\quad \begin{pmatrix} y \\ z \end{pmatrix}_{x=0} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \longrightarrow \begin{pmatrix} y \\ z \end{pmatrix} = \underset{\sim}{u}$

$x \in [0, 2]$, $h = 0.5$

$f(t, \underset{\sim}{u}) = \begin{pmatrix} -0.5u_1 \\ 4-0.3u_2-0.1u_1 \end{pmatrix}$ VECTOR DE DOS VARIABLES

A) EULER

B) RK4



SOLUCIONES APROXIMADAS

A) EULER

$\underset{\sim}{u}_{i+1} = \underset{\sim}{u}_i + h\, f(x_i, \underset{\sim}{u}_i)$

- $x_0 = 0$: $\underset{\sim}{u}_0 = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$
- $x_1 = 0.5$: $\underset{\sim}{u}_1 = \underset{\sim}{u}_0 + h\cdot f(x_0, \underset{\sim}{u}_0) = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + 0.5\cdot f\left(0, \begin{pmatrix} 4 \\ 6 \end{pmatrix}\right) = \begin{pmatrix} 4 \\ 6 \end{pmatrix} + 0.5\cdot\begin{pmatrix} -2 \\ 2.8 \end{pmatrix} = \begin{pmatrix} 3 \\ 6.9 \end{pmatrix}$
- $x_2 = 1$: $\underset{\sim}{u}_2 = \underset{\sim}{u}_1 + h\cdot f(x_1, \underset{\sim}{u}_1) = \begin{pmatrix} 3 \\ 6.9 \end{pmatrix} + 0.5\cdot f\left(0.5, \begin{pmatrix} 3 \\ 6.9 \end{pmatrix}\right) = \begin{pmatrix} 3 \\ 6.9 \end{pmatrix} + 0.5\cdot\begin{pmatrix} -1.5 \\ 1.63 \end{pmatrix} = \begin{pmatrix} 2.25 \\ 7.715 \end{pmatrix}$
- $x_3 = 1.5$: $\underset{\sim}{u}_3 = \underset{\sim}{u}_2 + h\cdot f(x_2, \underset{\sim}{u}_2) = \begin{pmatrix} 1.6875 \\ 8.44525 \end{pmatrix}$
- $x_4 = 2$: $\underset{\sim}{u}_4 = \underset{\sim}{u}_3 + h\cdot f(x_3, \underset{\sim}{u}_3) = \begin{pmatrix} 1.265625 \\ 9.09408 \end{pmatrix}$

B) RK4

OTRA VERSIÓN DE LA FORMULA (la h está fuera)

- $\underset{\sim}{k_1} = f(x_i, \underset{\sim}{u_i}) \cdot h$
- $\underset{\sim}{k_2} = f\left(x_i + \frac{h}{2}, \underset{\sim}{u_i} + \frac{\underset{\sim}{k_1}}{2}\right) \cdot h$
- $\underset{\sim}{k_3} = f\left(x_i + \frac{h}{2}, \underset{\sim}{u_i} + \frac{\underset{\sim}{k_2}}{2}\right) \cdot h$
- $\underset{\sim}{k_4} = f(x_i + h, \underset{\sim}{u_i} + \underset{\sim}{k_3}) \cdot h$

$$\underset{\sim}{u_{i+1}} = \underset{\sim}{u_i} + \frac{\underset{\sim}{k_1} + 2\underset{\sim}{k_2} + 2\underset{\sim}{k_3} + \underset{\sim}{k_4}}{6}$$

- $x_0 = 0$: $\underset{\sim}{u_0} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

- $x_1 = 0{,}5$: (1) $\underset{\sim}{k_1} = f(x_0, \underset{\sim}{u_0}) \cdot h = f\left(0, \begin{pmatrix} 4 \\ 6 \end{pmatrix}\right) 0{,}5 = \begin{pmatrix} -1 \\ 0{,}9 \end{pmatrix}$

(2) $\underset{\sim}{k_2} = f\left(x_0 + \frac{h}{2}, \underset{\sim}{u_0} + \frac{\underset{\sim}{k_1}}{2}\right) \cdot h = f\left(0{,}25; \begin{matrix} 3{,}5 \\ 6{,}45 \end{matrix}\right) \cdot 0{,}5 = f\begin{pmatrix} -0{,}5 \cdot 3{,}5 \\ 4 - 0{,}3 \cdot 6{,}45 - 0{,}1 \cdot 3{,}5 \end{pmatrix} 0{,}5 =$

$= \begin{pmatrix} -0{,}875 \\ 0{,}875 \end{pmatrix}$

(3) $\underset{\sim}{k_3} = f\left(x_0 + \frac{h}{2}, u_0 + \frac{\underset{\sim}{k_2}}{2}\right) \cdot h = f\left(0{,}25, \begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -0{,}4375 \\ 0{,}4375 \end{pmatrix}\right) \cdot 0{,}5 = \begin{pmatrix} -0{,}890625 \\ 0{,}8575625 \end{pmatrix}$

(4) $\underset{\sim}{k_4} = f(x_0 + h, \underset{\sim}{u_0} + \underset{\sim}{k_3}) \cdot h = \begin{pmatrix} -0{,}777343 \\ 0{,}815897 \end{pmatrix}$

$$\underset{\sim}{u_1} = \underset{\sim}{u_0} + \frac{\underset{\sim}{k_1} + 2\underset{\sim}{k_2} + 2\underset{\sim}{k_3} + \underset{\sim}{k_4}}{6} = \begin{pmatrix} 3{,}115234 \\ 6{,}857670 \end{pmatrix}$$

MÁS EXACTO QUE EULER

PERO MUCHO MÁS LENTO

ECUACIÓN DIFERENCIAL DE ORDEN SUPERIOR

- Se transforma en un sistema de ecuaciones diferenciales

$y''' + xy' \cdot y = 0; \quad y(1) = 0, \quad y'(1) = 0, \quad y''(1) = 1$

Solución aproximada en $x = 1.1$ con $\boxed{\text{RK4}}$

CAMBIO:

$$\left.\begin{aligned} u_1 &= y \\ u_2 &= y' \\ u_3 &= y'' \end{aligned}\right\} \longrightarrow \left.\begin{aligned} u_1' &= u_2 \\ u_2' &= u_3 \\ y''' = u_3' &= -x\, u_2 \cdot u_1 \end{aligned}\right\}$$

(DE LA ECUACIÓN DEL ENUNCIADO)

HASTAPEN BALDINTZA

$$\underset{\sim}{u_0} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{matrix} \to u_1 \\ \to u_2 \\ \to u_3 \end{matrix}$$

$$\begin{pmatrix} u_1' \\ u_2' \\ u_3' \end{pmatrix} = \boxed{\begin{pmatrix} u_2 \\ u_3 \\ -x\,u_2 \cdot u_1 \end{pmatrix}} \; f$$

$h = 0,1$

$x_0 = 1$, $x_1 = 1,1$

- $x_1 = 1,1$

(1) $\underset{\sim}{k_1} = f(x_0, \underset{\sim}{u_0}) = f\left(1, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

(2) $\underset{\sim}{k_2} = f\left(x_0 + \frac{h}{2}, \underset{\sim}{u_0} + \frac{k_1 \cdot h}{2}\right) = f\left(1,05, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0,05 \\ 0 \end{pmatrix}\right) = f\left(1,05, \begin{pmatrix} 0 \\ 0,05 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} 0,05 \\ 1 \\ 0 \end{pmatrix}$

(3) $\underset{\sim}{k_3} = f\left(x_0 + \frac{h}{2}, \underset{\sim}{u_0} + \frac{k_2 \cdot h}{2}\right) = \begin{pmatrix} 0,05 \\ 1 \\ -0,00013125 \end{pmatrix}$

(4) $\underset{\sim}{k_4} = f\left(x_0 + h, \underset{\sim}{u_0} + \underset{\sim}{k_3} \cdot h\right) = f\left(1,2, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0,05 \\ 1 \\ -0,00013125 \end{pmatrix} \cdot 0,1\right) = \begin{pmatrix} 0,1 \\ 0,999987 \\ -0,00055 \end{pmatrix}$

$\hookrightarrow \underset{\sim}{u_1} = \underset{\sim}{u_0} + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6} \cdot h = \begin{pmatrix} 0,005 \\ \sim \\ \sim \end{pmatrix}$ — $u_1 = y$, $u_2 = y'$, $u_3 = y''$

$$\boxed{y(1,1) = 0,005}$$

Como nos piden la solución aproximada de $x = 1,1$, solo necesitamos el valor de la "y". Por lo tanto, no necesitaríamos el valor de "y'" y "y''". Si nos pidieran el vector entero, sí que tendríamos que dar los tres valores.

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① $y(x) = C_1 \cdot e^x + C_2 x \cdot e^x + C_3 e^{2x} + C_4 x \cdot e^{2x}$, $\quad C_i \in \mathbb{R}$, $i = 1,2,3,4$

A) Orden 4

B) Sol: $y_1 = e^x$, $y_2 = x \cdot e^x \longrightarrow R = 1$ DOBLE

$y_3 = e^{2x}$, $y_4 = x \cdot e^{2x} \longrightarrow R = 2$ DOBLE

Ecuación característica: $(R-1)^2 \cdot (R-2)^2 = 0 \longrightarrow (R^2 - 2R + 1) \cdot (R^2 - 4R + 4) = 0$

$R^4 - 6R^3 + 13R^2 - 12R + 4 = 0$

$\hookrightarrow$ **EDA**: $y^{IV} - 6y''' + 13y'' - 12y' + 4y = 0$