ADIB.

$$\begin{cases} u''' + u'' - tv' + v = t \\ v'' + u + u' = 8 \end{cases}$$

3 hastapen baldintza
3. ordenekoa delako

$$\begin{cases} u(0) = 1 \\ u'(0) = 2 \\ u''(0) = 3 \end{cases}$$

$$\begin{cases} v(0) = -1 \\ v'(0) = -2 \end{cases}$$

2 hastapen baldintza
2. ordenekoa delako

$$x_1 = u \xrightarrow{\text{DERIBATUA}} x_1' = u' = x_2$$

$$x_2 = u' \longrightarrow x_2' = u'' = x_3$$

$$x_3 = u'' \longrightarrow x_3' = u''' = t - u'' + tv' - v = t - x_3 + x_5 t - x_4$$

$$x_4 = v \longrightarrow x_4' = v' = x_5$$

$$x_5 = v' \longrightarrow x_5' = v'' = 8 - u - u' = 8 - x_1 - x_2$$

$$\underset{\sim}{x}'(t) = \begin{pmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \\ x_4'(t) \\ x_5'(t) \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & t \\ 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 & 0 \end{pmatrix}}_{P(t)} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ t \\ 0 \\ 8 \end{pmatrix}$$

$$\boxed{\underset{\sim}{x}'(t) = P(t) \cdot \underset{\sim}{x}(t) + \underset{\sim}{q}(t)}$$

HASTAPEN BALDINTZAKO PROBLEMA

$$x(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \\ x_5(0) \end{pmatrix} = \begin{pmatrix} u(0) \\ u'(0) \\ u''(0) \\ v(0) \\ v'(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ -1 \\ -2 \end{pmatrix}$$

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ADIB

SISTEMA ERAN PLANTEATU

$$\underset{\sim}{X}'(t) = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \cdot \underset{\sim}{X}(t) \quad \text{NON} \left\{ \begin{array}{l} A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \\ \underset{\sim}{X}(t) = \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} \text{ NON } \left\{ \begin{array}{l} u' = u + v \\ v' = 4u + v \end{array} \right. \end{array} \right.$$

$$\underset{\sim}{X}'(t) = A \cdot \underset{\sim}{X}(t)$$

1 AUTOBALIOAK KALKULATU

POLINOMIO KARAKTERISTIKOA ←

$$p_A(t) = |A - tI_n| = \begin{vmatrix} 1-t & 1 \\ 4 & 1-t \end{vmatrix} = (1-t)^2 - 4 = (1-t)^2 - 2^2 =$$

$$(1-t-2) \cdot (1-t+2) = (-1-t) \cdot (3-t)$$

AUTOBALIOAK $\longrightarrow$ $Spec(A) = \{-1, 3\}$ $\{$ SINPLEAK (ez dira errepikatzen)

2 AUTOBEKTOREAK $(\underset{\sim}{X}_1, \underset{\sim}{X}_2)$

polinomio karakteristikoa bere autobalioa ordezkatuta

3 AUTOBALIOA • $V(\lambda) = V(3) = Ker \begin{pmatrix} 1-3 & 1 \\ 4 & 1-3 \end{pmatrix} = Ker \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix}$

$$\underset{\sim}{V} = \begin{pmatrix} x \\ y \end{pmatrix} \in V(3) \Longleftrightarrow \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

↓ SISTEMA ERAN IDATZI

$$\left\{ \begin{array}{l} -2x + y = 0 \\ 4x - 2y = 0 \end{array} \right. \qquad \begin{pmatrix} -2 & 1 \\ 4 & -2 \end{pmatrix}$$

- dim = 2
- heina = 1 (determinantea nulua da, $\lambda=3$ ordezkatu baitugu)

$$\left\{ \begin{array}{l} x = x \\ y = 2x \end{array} \right. \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2x \end{pmatrix} = x \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$2-1 = 1$ autobektore lortuko da $\lambda=3$ autobalioarekin

$$V(3) = \left\langle \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\rangle = \mathbb{R} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = Span\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\} = \left\{ t \begin{pmatrix} 1 \\ 2 \end{pmatrix} \mid t \in \mathbb{R} \right\}$$

↳ AZPIESPAZIOA

$$\boxed{\underset{\sim}{X}_1(t) = e^{\lambda t} \langle V(\lambda_1) \rangle = e^{3t} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} e^{3t} \\ 2e^{3t} \end{pmatrix}} \quad \underset{\sim}{X}_1 \text{ AUTOBEKTOREA}$$

Azpiespazioa

-1 AUTOBALIOA • $V(\lambda) = V(-1) = \ker \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$

$$\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix} \in V(-1) \longleftrightarrow \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

↓ SISTEMA ERAN IDATZI

$$\begin{cases} 2x + y = 0 \\ 4x + 2y = 0 \end{cases} \qquad \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} \quad \begin{array}{l} \dim = 2 \\ \underline{\text{heina} = 1} \\ \text{dimKer} = 1 \end{array}$$

↳ $\lambda = -1$ autobalioari dagokion autobektore bakarra lortu dugu.

$$\begin{cases} x = x \\ y = -2x \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -2x \end{pmatrix} = x \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$V(-1) = \operatorname{Span}\left\{\begin{pmatrix} 1 \\ -2 \end{pmatrix}\right\} = \left\{ t \begin{pmatrix} 1 \\ -2 \end{pmatrix} \mid t \in \mathbb{R} \right\}$$

$$\boxed{\underset{\sim}{X_2} = e^{-t} \underbrace{\begin{pmatrix} 1 \\ -2 \end{pmatrix}}_{\text{Lotutako autobalioa}} = \begin{pmatrix} e^{-t} \\ -2e^{-t} \end{pmatrix}} \qquad \underset{\sim}{X_2} \text{ AUTOBEKTOREA}$$

FUNTSEZKO OINARRIZKO MATRIZEA $\leftarrow$ $\mathcal{X} = \left(\underset{\sim}{X_1} \mid \underset{\sim}{X_2} \right)$ IZANIK, $\mathcal{X}(t) = \begin{pmatrix} e^{3t} & e^{-t} \\ 2e^{3t} & -2e^{-t} \end{pmatrix}$

$$X_{SO}(t) = \mathcal{X}(t) \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = c_1 \underset{\sim}{X_1}(t) + c_2 \underset{\sim}{X_2}(t) = \begin{pmatrix} c_1 e^{3t} + c_2 e^{-t} \\ 2c_1 e^{3t} - 2c_2 e^{-t} \end{pmatrix}$$

↓ HORTAZ SISTEMA ERAN PLANTEATUTA

$$\begin{cases} u(t) = c_1 e^{3t} + c_2 e^{-t} \\ v(t) = 2c_1 e^{3t} - 2c_2 e^{-t} \end{cases} \quad \forall c_1, c_2 \in \mathbb{R} \qquad X(t) = \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} \text{ BAITA}$$

$X_{SO} \longrightarrow$: SISTEMAREN SOLUZIO OROKORRA

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1

①

$$\underset{\sim}{X}_1(t) = \begin{pmatrix} t \\ 1 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_1' \end{pmatrix} \quad \underset{\sim}{X}_2(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix} = \begin{pmatrix} y_2 \\ y_2' \end{pmatrix}$$

A) WRONSKIARRA

$$W_{y_1,y_2} = \left| \underset{\sim}{X}_1(t) \quad \underset{\sim}{X}_2(t) \right| = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix} = 2t^2 - t^2 = t^2$$

B) LINEALKI INDEPENDIENTEAK ZE TARTETAN?

Determinantea planteatu $\longrightarrow$ t-ren balioa lortu det = 0 denerako

$$\begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix} = 2t^2 - t^2 = t^2 = 0 \,; \quad \boxed{t = 0}$$

Hau da, $t = 0$ denean $\underset{\sim}{X}_1(t)$ eta $\underset{\sim}{X}_2(t)$ ez dira linealki independienteak izango

↓ HORTAZ

$\underset{\sim}{X}_1(t)$ ETA $\underset{\sim}{X}_2(t)$ LINEALKI INDEPENDIENTEAK $(-\infty, 0) \cup (0, \infty)$

C) 1. ordenako EDA NON $\underset{\sim}{X}_1$ eta $\underset{\sim}{X}_2$ soluzioak

$\underset{\sim}{X}_1 = \begin{pmatrix} t \\ 1 \end{pmatrix}$ sistemaren soluzioetako bat non eta

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\underset{\sim}{X}_1' = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} t \\ 1 \end{pmatrix} ; \quad \begin{cases} at + b = 1 \,; & b = 1 - at \\ ct + d = 0 \,; & d = -ct \end{cases}$$

$$\underset{\sim}{X}_2 = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$$

$$\underset{\sim}{X}_2' = \begin{pmatrix} 2t \\ 2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} t^2 \\ 2t \end{pmatrix} \quad \begin{cases} at^2 + 2tb = 2t \\ ct^2 + 2dt = 2 \end{cases}$$

- $at^2 + 2t(1 - at) = 2t \,; \quad at^2 + 2t - 2at^2 = 2t \,; \quad -at^2 = 0 \,; \quad \boxed{a = 0}$
  - $\hookrightarrow 0 \cdot t + b = 1 \,; \quad \boxed{b = 1}$
- $ct^2 + 2t(-ct) = 2 \,; \quad ct^2 - 2ct^2 = 2 \,; \quad -ct^2 = 2 \,; \quad \boxed{c = -2/t^2}$
  - $\hookrightarrow \boxed{d = 2/t}$

$$\underset{\sim}{X}' = \begin{pmatrix} 0 & 1 \\ -2/t^2 & 2/t \end{pmatrix} \underset{\sim}{X}$$

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2

2

A) $\underset{\sim}{x}' = \underbrace{\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}}_{A} \underset{\sim}{x}$

1 AUTOBALIOAK KALKULATU

$P_A(t) = |A - tI_n| = \begin{vmatrix} 3-t & -2 \\ 2 & -2-t \end{vmatrix} = (3-t)\cdot(-2-t) - (-2)\cdot 2 =$

$= -6 - 3t + 2t + t^2 + 4 = t^2 - t - 2$

$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\cdot 1\cdot(-2)}}{2\cdot 1} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2}$ → $\boxed{t = 2}$, $\boxed{t = -1}$ SINPLEAK

$Spec(A) = \{2, -1\}$

2 AUTOBEKTOREAK

$\boxed{\lambda = 2}$

$\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix} \in V(2) \longleftrightarrow \begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

dim = 2
heina = 1
dimker = 1
AUTOBALIO BAKARRA

$x - 2y = 0 \longrightarrow \begin{cases} x = 2y \\ y = y \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2y \\ y \end{pmatrix} = y\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\boxed{\underset{\sim}{X}_1(t) = e^{2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}}$

$\boxed{\lambda = -1}$

$\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix} \in V(-1) \longleftrightarrow \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

dim = 2
heina = 1
dimker = 1
AUTOBALIO BAKARRA

$2x - y = 0 \longrightarrow \begin{cases} x = x \\ y = 2x \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2x \end{pmatrix} = x\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\boxed{\underset{\sim}{X}_2(t) = e^{-t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}}$

FUNTSEZKO MATRIZEA $X(t) = \begin{pmatrix} 2e^{2t} & e^{-t} \\ e^{2t} & 2e^{-t} \end{pmatrix}$

$\underset{\sim}{X}_{sol}(t) = X(t)\cdot\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 2C_1 e^{2t} + C_2 e^{-t} \\ C_1 e^{2t} + 2C_2 e^{-t} \end{pmatrix}$

B)

B) $\underset{\sim}{x}' = \underbrace{\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}}_{A} \underset{\sim}{x}$

① AUTOBALIOAK

$P_A(t) = |A - tI| = \begin{vmatrix} 1-t & i \\ -i & 1-t \end{vmatrix} = (1-t)^2 + i^2 = 1 + t^2 - 2t - 1 =$

$= t^2 - 2t = t(t-2) \longrightarrow \boxed{t=0}$, $\boxed{t=2}$ SINPLEAK

$Spec(A) = \{0, 2\}$

② AUTOBEKTOREAK

$\boxed{\lambda = 0}$

$\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix} \in V(0) \longleftrightarrow \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ dim = 2, hewo = 1, dimker = 1

$x + iy = 0 \longrightarrow \begin{cases} x = -iy \\ y = y \end{cases} \rightsquigarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -iy \\ y \end{pmatrix} = y \begin{pmatrix} -i \\ 1 \end{pmatrix}$

$\boxed{\underset{\sim}{X}_1 = e^{0t} \begin{pmatrix} -i \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 1 \end{pmatrix}}$

$\boxed{\lambda = 2}$

$\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix} \in V(2) \longleftrightarrow \begin{pmatrix} -1 & i \\ -i & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ dim = 2, hewo = 1, dimker = 1

$-x + iy = 0 \longrightarrow \begin{cases} x = iy \\ y = y \end{cases} \rightsquigarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} iy \\ y \end{pmatrix} = y \begin{pmatrix} i \\ 1 \end{pmatrix}$

$\boxed{\underset{\sim}{X}_2 = e^{2t} \begin{pmatrix} i \\ 1 \end{pmatrix}}$

ITSEZKO
BIZKA

$X(t) = \begin{pmatrix} -i & e^{2t} i \\ 1 & e^{2t} \end{pmatrix}$

$\underset{\sim}{X}_{SO}(t) = X(t) \cdot \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} -ic_1 + c_2 e^{2t} i \\ c_1 + c_2 e^{2t} \end{pmatrix}$

c)

$$\underset{\sim}{x}' = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix} \underset{\sim}{x}$$

① AUTOBALIOAK

$$P_A(t) = |A - tI| = \begin{vmatrix} 1-t & 1 & 2 \\ 1 & 2-t & 1 \\ 2 & 1 & 1-t \end{vmatrix} = \left[(1-t)^2(2-t)+4\right] - \left[4(2-t)+(1-t)+(1-t)\right] =$$

$$= (1+t^2-2t)(2-t)+4-8+4t-1+t-1+t = 2-t+2t^2-t^3-4t+2t^2-6+6t =$$

$$= -t^3+4t^2+t-4$$

$$\begin{array}{c|cccc} & -1 & 4 & 1 & -4 \\ 1 & & -1 & 3 & 4 \\ \hline & -1 & 3 & 4 & \cancel{0} \end{array}$$

$$\hookrightarrow -t^2+3t+4=0 \longrightarrow t = \frac{-3 \pm \sqrt{3^2-4(-1)\cdot 4}}{2\cdot(-1)} = \frac{-3\pm 5}{-2} \quad \begin{array}{l} \nearrow \boxed{t=4} \\ \searrow \boxed{t=-1} \end{array}$$

$\boxed{t=1}$

$$Spec(A) = \{4, -1\}$$

② AUTOBEKTOREAK

$\boxed{\lambda=4}$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V(4) \longleftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} -3 & 1 & 2 \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\dfrac{\begin{array}{l} dim = 3 \\ heina = 2 \end{array}}{dimker = 1}$

$$x-2y+z=0 \longrightarrow \begin{cases} x = y \\ y = y \\ z = y \end{cases} \hookrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y \\ y \\ y \end{pmatrix} = y \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\boxed{\underset{\sim}{X}_1 = e^{4t}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}$$

$\boxed{\lambda=-1}$

$$\underset{\sim}{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V(-1) \longleftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\dfrac{\begin{array}{l} dim = 3 \\ heina = 2 \end{array}}{dimker = 1}$

$$2x+y+2z=0 \longrightarrow \begin{cases} x = 0 \\ y = z \\ z = -2z \end{cases} \hookrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ z \\ 2z \end{pmatrix} = z \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$\boxed{\underset{\sim}{X}_2 = e^{-t}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}}$$

$\boxed{\lambda=1}$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V(1) \longleftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\dfrac{\begin{array}{l} dim = 3 \\ heina = 2 \end{array}}{dimker = 1}$

$$y+2z=0 \longrightarrow \begin{cases} x = 0 \\ y = -2z \\ z = z \end{cases} \hookrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -2z \\ z \end{pmatrix} = z \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

$$\boxed{\underset{\sim}{X}_3 = e^{t}\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}}$$

$$x_{so}(t) = C_1 e^{4t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + C_2\, e^{-t} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + C_3\, e^{t} \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

ADIB.

$$\underset{\sim}{x}' = \underbrace{\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}}_{A} \underset{\sim}{x} \qquad \boxed{\underset{\sim}{x}' = A\underset{\sim}{x}}$$

1 AUTOBALIOAK AURKITU

OLINOMIO $\longrightarrow$ $P_A(t) = |A - tI_n| = \begin{vmatrix} -t & 1 & 1 \\ 1 & -t & 1 \\ 1 & 1 & -t \end{vmatrix} = -t^3 + 2 + 3t$

Maila handieneko osagaiaren koefizientea (kasu honetan -1) kontuan hartu behar dugu

RUFFINI

$$\begin{array}{c|cccc} & -1 & 0 & 3 & 2 \\ -1 & & 1 & -1 & -2 \\ \hline & -1 & 1 & 2 & \boxed{0} \\ 2 & & -2 & -2 & \\ \hline & -1 & -1 & \boxed{0} & \end{array} \longrightarrow -t-1=0;\ \boxed{t=-1}$$

$-t^3 + 3t + 2 = -(t+1)^2 \cdot (t-2)$ $\{$ $\mathrm{Spec}(A) = \{2, -1\}$

BIKOITZA

SINPLEA

2 AUTOBEKTOREAK ATERA

$\boxed{\lambda = 2}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V(2) \longleftrightarrow \underbrace{\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}}_{P_A(2)} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{array}{r} \text{dim} = 3 \\ \ominus\ \text{heina} = 2 \\ \hline \text{dimKer} = 1 \end{array}$$

$\lambda = 2$ autobektore bakarra lortuko dugu.

SISTEMA ERAN BERRIDATZI

$$\begin{cases} -2x + y + z = 0 \\ x - 2y + z = 0 \\ x + y - 2z = 0 \end{cases}$$

$$\boxed{x = y = z} \longrightarrow \begin{cases} x = x \\ y = x \\ z = x \end{cases} \longrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ x \\ x \end{pmatrix} = x \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$V(2) = \mathrm{Span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

$$\underset{\sim}{X_1}(t) = e^{2t}\, V(2) = e^{2t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\underset{\sim}{X_1}$: AUTOBEKTOREA

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$\boxed{\lambda = -1}$ Autobektore bikoitza, hemendik 2 autobektore lortuko ditugu.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V(-1) \longleftrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left\{ \begin{array}{l} \dim = 3 \\ \underline{\text{heina} = 1} \\ \dim \ker = 2 \end{array} \right. \rightarrow \text{[illegible]}$$

SISTEMA ERAN JARRI

$$\{ x + y + z = 0 \longrightarrow \left\{ \begin{array}{l} x = x \\ y = y \\ z = -x - y \end{array} \right. \longrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ -x-y \end{pmatrix} = x \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + y \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

2 daudelako

$$V^{(2)}(-1) = \operatorname{Span} \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\}$$

$$\left. \begin{array}{l} \underset{\sim}{x_2}(t) = e^{-t} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \\ \underset{\sim}{x_3}(t) = e^{-t} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \end{array} \right\} \text{2 AUTOBEKTORE}$$

$$X(t) = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} e^{2t} & e^{-t} & 0 \\ e^{2t} & 0 & e^{-t} \\ e^{2t} & -e^{-t} & -e^{-t} \end{pmatrix} \end{array} \quad \text{FUNTSEZKO MATRIZEA}$$

- $$\underset{\sim}{x_{so}}(t) = X(t) \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = c_1 \underset{\sim}{x_1}(t) + c_2 \underset{\sim}{x_2}(t) + c_3 \underset{\sim}{x_3}(t) =$$

$$= \begin{pmatrix} c_1 e^{2t} + c_2 e^{-t} \\ c_1 e^{2t} + c_3 e^{-t} \\ c_1 e^{2t} - c_2 e^{-t} - c_3 e^{-t} \end{pmatrix} \quad \forall c_1, c_2, c_3 \in \mathbb{R}$$

SISTEMAREN SOLUZIO OROKORRA

4

4. $r=2$ ERRO HIRUKOITZA?

$$\underset{\sim}{x}' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix} \underset{\sim}{x}$$

1. AUTOBALIOAK

$$P_A(t) = |A - tI_3| = \begin{vmatrix} 1-t & 1 & 1 \\ 2 & 1-t & -1 \\ -3 & 2 & 4-t \end{vmatrix} = \left[(1-t)^2(4-t)+3+4\right] - \left[-3(1-t)-2(1-t)+2(4-t)\right] =$$

$$= \left[(1+t^2-2t)\cdot(4-t)+7\right] - \left[-3+3t-2+2t+8-2t\right] =$$

$$= \left[4+4t^2-8t-t-t^3+2t^2+7\right] - \left[3+3t\right] = -t^3+6t^2-12t+8$$

RUFFINI

	-1	6	-12	8
2		-2	8	-8
	-1	4	-4	$\boxed{0}$
2		-2	4	
	-1	2	$\boxed{0}$	
2		-2		
	-1	$\boxed{0}$		

$\longrightarrow$ HORTAZ,

$$-t^3+6t^2-12t+8 = -(t-2)^3 \longrightarrow \boxed{r=2}$$

HIRUKOITZA

2. AUTOBEKTOREAK

- $V(2)$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V(2) \longleftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \underbrace{\begin{pmatrix} -1 & 1 & 1 \\ 2 & -1 & -1 \\ -3 & 2 & 2 \end{pmatrix}}_{(A-2I)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{array}{l} \dim = 3 \\ \text{heina} = 2 \\ \hline \dim\ker = 1 \end{array}$$

$$\left\{ -x+y+z=0 \right. \longrightarrow \left\{ \begin{array}{l} x=0 \\ y=y \\ z=-y \end{array} \right. \leadsto \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ y \\ -y \end{pmatrix} = y\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\boxed{\underset{\sim}{x}_1 = e^{2t}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}}$$

- $V^2(2)$

$$(A-2I)^2 = (A-2I)\cdot(A-2I) = \begin{pmatrix} -1 & 1 & 1 \\ 2 & -1 & -1 \\ -3 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 & 1 \\ 2 & -1 & -1 \\ -3 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V^2(2) \longleftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{array}{l} \dim = 3 \\ \text{heina} = 1 \\ \hline \dim\ker = 2 \end{array}$$

$$-x+y+z=0 \longrightarrow \left\{ \begin{array}{l} x=y+z \\ y=y \\ z=z \end{array} \right. \leadsto \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y+z \\ y \\ z \end{pmatrix} = y\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\underset{\sim}{X_2}(t) = e^{2t}\left[\underset{\sim}{V_2} + t(A-2I)\underset{\sim}{V_2}\right] = e^{2t}\left[\underset{3\times1}{\begin{pmatrix}1\\1\\0\end{pmatrix}} + t\underset{3\times3}{\begin{pmatrix}-1 & 1 & 1\\ 2 & -1 & -1\\ -3 & 2 & 2\end{pmatrix}}\underset{3\times1}{\begin{pmatrix}1\\1\\0\end{pmatrix}}\right] =$$

$$= e^{2t}\left[\begin{pmatrix}1\\1\\0\end{pmatrix} + \begin{pmatrix}0\\t\\-t\end{pmatrix}\right] = e^{2t}\begin{pmatrix}1\\1+t\\-t\end{pmatrix}$$

identitate naborrendua zutabea

- $V^3(2) = \mathbb{R}^3 = \text{Span}\left\{\begin{pmatrix}0\\1\\-1\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$ 3 zutabeak linealki independienteak

$$\underset{\sim}{X_3}(t) = e^{2t}\left[\underset{\sim}{V_3} + t(A-2I)\underset{\sim}{V_3} + \frac{t^2}{2}(A-2I)^2\underset{\sim}{V_3}\right] =$$

$$= e^{2t}\left[\begin{pmatrix}0\\0\\1\end{pmatrix} + \begin{pmatrix}-t & t & t\\ 2t & -t & -t\\ -3t & 2t & 2t\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix} + \begin{pmatrix}-t^2/2 & t^2/2 & t^2/2\\ t^2 & -t^2/2 & -t^2/2\\ -3t^2/2 & t^2 & t^2\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right] =$$

$$= e^{2t}\left[\begin{pmatrix}0\\0\\1\end{pmatrix} + \begin{pmatrix}t\\-t\\2t\end{pmatrix} + \begin{pmatrix}t^2/2\\-t^2/2\\t^2\end{pmatrix}\right] = e^{2t}\begin{pmatrix}t+t^2/2\\-t-t^2/2\\1+2t+t^2\end{pmatrix}$$

$$X_{30}(t) = C_1 e^{2t}\begin{pmatrix}0\\1\\-1\end{pmatrix} + C_2 e^{2t}\begin{pmatrix}1\\1+t\\-t\end{pmatrix} + C_3 e^{2t}\begin{pmatrix}t+t^2/2\\-t-t^2/2\\1+2t+t^2\end{pmatrix}$$

B) $\underset{\sim}{X}' = \begin{pmatrix}5 & -3 & -2\\ 8 & -5 & -4\\ -4 & 3 & 3\end{pmatrix}\cdot\underset{\sim}{X}$

① AUTOBALIOAK

$$P_A(t) = |A-tI| = \begin{vmatrix}5-t & -3 & -2\\ 8 & -5-t & -4\\ -4 & 3 & 3-t\end{vmatrix} = \left[(5-t)(-5-t)(3-t)-48-48\right] - \left[8(-5-t)-12(5-t)-24(3-t)\right]$$

$$= \left[(-25-\cancel{5t}+\cancel{5t}+t^2)(3-t)-96\right] - \left[-40-8t-60+12t-72+24t\right] =$$

$$= \left[-75+25t+3t^2-t^3-96\right] - \left[28t-172\right] = -t^3+3t^2-3t+1$$

$+172$
-171

RUFFINI

	-1	3	-3	1
1		-1	2	-1
	-1	2	-1	$\boxed{0}$
1		-1	1	
	-1	1	$\boxed{0}$	
1		-1		
	-1	$\boxed{0}$		

HORTAZ, $\boxed{r=1}$ ERRO HIRUKOITZA

② AUTOBEKTOREAK

- $V(1)$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 4 & -3 & -2 \\ 8 & -6 & -4 \\ -4 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{array}{l} \dim = 3 \\ \underline{\text{heina} = 1} \\ \text{dimker} = 2 \end{array}$$

$$4x - 3y - 2z = 0 \longrightarrow \begin{cases} x = \frac{3y}{4} + \frac{z}{2} \\ y = y \\ z = z \end{cases} \longrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{3y}{4} + \frac{z}{2} \\ y \\ z \end{pmatrix} = y \begin{pmatrix} 3/4 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix}$$

$$\boxed{\underset{\sim}{x_1} = e^t \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix}} \qquad \boxed{\underset{\sim}{x_2} = e^t \begin{pmatrix} 3/4 \\ 1 \\ 0 \end{pmatrix}}$$

$\mathbb{R}^3$-ko bektorea $\longrightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\begin{vmatrix} 1/2 & 3/4 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1/2 \neq 0$$

$$\underset{\sim}{x_3} = e^t \left(\underset{\sim}{v_3} + t(A - 2I)\underset{\sim}{v_3} \right) = e^t \left[\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 4t & -3t & -2t \\ 8t & -6t & -4t \\ -4t & 3t & 2t \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right] =$$

$$= e^t \left[\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -2t \\ -4t \\ 2t \end{pmatrix} \right] = e^t \begin{pmatrix} -2t \\ -4t \\ 1+2t \end{pmatrix}$$

$$x_{so}(t) = C_1 e^t \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix} + C_2 e^t \begin{pmatrix} 3/4 \\ 1 \\ 0 \end{pmatrix} + C_3 e^t \begin{pmatrix} -2t \\ -4t \\ 1+2t \end{pmatrix}$$

7 7)

A) $\underset{\sim}{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \underset{\sim}{x}$

① AUTOBALIOAK

$$P_A(t) = |A - tI| = \begin{vmatrix} 3-t & -2 \\ 2 & -2-t \end{vmatrix} = (3-t)(-2-t) + 4 = -6 - 3t + 2t + t^2 + 4 =$$

$$= t^2 - t - 2 \rightarrow t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{1 \pm 3}{2} \rightarrow \boxed{t = 2}, \ \boxed{t = -1}$$

$\text{Spec}(A) = \{2, -1\}$

② AUTOBEKTOREAK

$\boxed{\lambda = 2}$

$$\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix} \in V(2) \longleftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{array}{l} \dim = 2 \\ \underline{\text{heina} = 1} \\ \text{dimker} = 1 \end{array}$$

$$x - 2y = 0 \longrightarrow \begin{cases} x = 2y \\ y = y \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2y \\ y \end{pmatrix} = y \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\boxed{\underset{\sim}{x_1} = e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}}$$

dim = 2
heina = 1
dim ker = 1

$\boxed{\lambda = -1}$

$$\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix} \in V(-1) \longleftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$2x - y = 0 \longrightarrow \begin{cases} x = x \\ y = 2x \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2x \end{pmatrix} = x \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\boxed{\underset{\sim}{\chi_2} = e^{-t} \begin{pmatrix} 1 \\ 2 \end{pmatrix}}$$

FUNTSEZKO MATRIZEA

$$X(t) = \begin{pmatrix} 2e^{2t} & e^{-t} \\ e^{2t} & 2e^{-t} \end{pmatrix}$$

$$\hookrightarrow \chi_{sol}(t) = X(t) \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 2C_1 e^{2t} + C_2 e^{-t} \\ C_1 e^{2t} + 2C_2 e^{-t} \end{pmatrix}$$

ADIB.

$$\begin{cases} x' = x - 5y \\ y' = x - 3y \end{cases} \longrightarrow \underset{\sim}{x}' = \begin{pmatrix} x' \\ y' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix}}_{A} \underset{\sim}{x} = A \cdot \underset{\sim}{x}$$

$\underset{\sim}{x} \rightarrow \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$

$x(0) = 1$
$y(0) = 1$ } HASTAPEN BALDINTZAK

① AUTOBALIOAK KALKULATU

POLINOMIO KARAKTERISTIKOA $\longrightarrow$ $P_A(t) = |A - tI_n| = \begin{vmatrix} 1-t & -5 \\ 1 & -3-t \end{vmatrix} = (1-t)\cdot(-3-t) + 5 = t^2 + 2t + 2$

$t^2 + 2t + 2 = 0$; $(t+1)^2 + 1 = 0$; $(t+1)^2 = -1$; $\sqrt{(t+1)^2} = \sqrt{-1}$;

$t + 1 = \pm i$; $\boxed{(t+1+i)\cdot(t+1-i) = 0}$

$Spe(A) = \{-1-i, -1+i\}$ AUTOBALIO KONPLEXUAK

② AUTOBEKTOREAK KALKULATU

$\boxed{\lambda = -1+i}$

$\begin{pmatrix} x \\ y \end{pmatrix} \in V(-1+i) \xrightarrow[\text{t-gatik}]{\text{PA-n ordezkatu}} \begin{pmatrix} 1+1-i & -5 \\ 1 & -3+1-i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 2-i & -5 \\ 1 & -2-i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\longrightarrow$ $\begin{cases} \dim = 2 \\ \text{heino} = 1 \\ \hline \text{dimker} = 1 \end{cases} \rightarrow$ autobektore 1

↓ SISTEMA ERAN JARRI

$$\begin{cases} (2-i)x - 5y = 0 \\ x - (2+i)y = 0 \end{cases} \longrightarrow \begin{cases} x = (2+i)y \\ y = y \end{cases} \rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (2+i)y \\ y \end{pmatrix} = y \begin{pmatrix} 2+i \\ 1 \end{pmatrix}$$

$$V(-1+i) = Span\left\{\begin{pmatrix} 2+i \\ 1 \end{pmatrix}\right\} = \left\{ t \begin{pmatrix} 2+i \\ 1 \end{pmatrix} \mid t \in \mathbb{R} \right\}$$

SOLUZIO KONPLEXUA

$$\underset{\sim}{X_1} = e^{\lambda t}\begin{pmatrix} 2+i \\ 1 \end{pmatrix} = e^{(-1+i)t}\begin{pmatrix} 2+i \\ 1 \end{pmatrix} = e^{-t}\cdot e^{it}\begin{pmatrix} 2+i \\ 1 \end{pmatrix} =$$

$$= e^{-t}\overbrace{(\cos t + i\sin t)}^{e^{it}}\begin{pmatrix} 2+i \\ 1 \end{pmatrix} = e^{-t}\begin{pmatrix} (\cos t + i\sin t)(2+i) \\ \cos t + i \sin t \end{pmatrix} =$$

hots soluzioa = 1

$$= e^{-t}\begin{pmatrix} 2\cos t + 2i\sin t + i\cos t - \sin t \\ \cos t + i\sin t \end{pmatrix} =$$

$$= \underbrace{\begin{pmatrix} e^{-t}(2\cos t - \sin t) \\ e^{-t}\cos t \end{pmatrix}}_{\text{ZATI ERREALA}} + i \underbrace{\begin{pmatrix} e^{-t}(\cos t + 2\sin t) \\ e^{-t}\sin t \end{pmatrix}}_{\text{ZATI IRUDIKARIA}}$$

$\underset{\sim}{X_1}$ AUTOBEKTORE BAT

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BESTE AUTOBALIOAREKIN PROZESU BERDINA JARRAITU

$\rightarrow \underset{\sim}{X_2} = \begin{pmatrix} e^{-t}(2\sin t + \cos t) \\ e^{-t}\sin t \end{pmatrix}$ $\underset{\sim}{X_2}$ AUTOBEKTOREA

$\downarrow$ HORTAZ, FUNTSEZKO MATRIZEA $\mathcal{X}(t) = \left(\underset{\sim}{X_1} \mid \underset{\sim}{X_2} \right)$

$$\mathcal{X}(t) = e^{-t} \begin{pmatrix} 2\cos t - \sin t & 2\sin t + \cos t \\ \cos t & \sin t \end{pmatrix}$$

$$X_{SO} = \underset{\sim}{X}(t) \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = e^{-t} \begin{pmatrix} C_1(2\cos t - \sin t) + C_2(2\sin t + \cos t) \\ C_1 \cos t + C_2 \sin t \end{pmatrix} \begin{matrix} x(t) \\ y(t) \end{matrix}$$

$\downarrow$ HASTAPEN BALDINTZAK ORDEZKATU

$$\underset{\sim}{X}(0) \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = e^{-0} \begin{pmatrix} 2\cos 0 - \sin 0 & 2\sin 0 + \cos 0 \\ \cos 0 & \sin 0 \end{pmatrix} \cdot \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{matrix} x(0) \\ y(0) \end{matrix}$$

$$\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \begin{cases} C_1 = 1 \\ C_2 = -1 \end{cases}$$

$$X_{SO} = e^{-t} \begin{pmatrix} 2\cos t - \sin t - 2\sin t - \cos t \\ \cos t - \sin t \end{pmatrix} = e^{-t} \begin{pmatrix} \cos t - 3\sin t \\ \cos t - \sin t \end{pmatrix} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$\rightarrow$ SISTEMAREN SOLUZIO OROKORRA

ADIB.

$$\underset{\sim}{X}' = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix}}_{A} \cdot \underset{\sim}{X}$$

① AUTOBALIOAK AURKITU

$$P_A(t) = |A - t I_n| = \begin{vmatrix} 1-t & 1 & 1 \\ 2 & 1-t & -1 \\ -3 & 2 & 4-t \end{vmatrix} = -(t-2)^3$$

$\mathrm{Spec}(A) = \{2\}$ HIRUKOITZA $(m=3)$

- $V(2)$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V(2) \longleftrightarrow \begin{pmatrix} -1 & 1 & 1 \\ 2 & -1 & -1 \\ -3 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\hookrightarrow$ dim = 3
heina = 2
dimker = 1 $\rightarrow$ autobektore bat

SISTEMA ERAN

$$\begin{cases} -x+y+z=0 \\ 2x-y-z=0 \end{cases} \longrightarrow \begin{cases} x=0 \\ y=y \\ z=-y \end{cases} \longrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ y \\ -y \end{pmatrix} = y\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$V(2) = \text{Span}\left\{\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\right\}$$

$$\underset{\sim}{X_1}(t) = e^{2t}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$ $\underset{\sim}{X_1}$ AUTOBEKTOREA

- $V^2(2)$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V^2(2) \longleftrightarrow \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \end{pmatrix}}^{(A-2I)^2} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\hookrightarrow$ dim = 3
heina = 1
dimker = 2 $\longrightarrow$ 2 AUTOBEKTORE

SISTEMA ERAN

$$\{ -x+y+z=0 \longrightarrow \begin{cases} x=y+z \\ y=y \\ z=z \end{cases} \longrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y+z \\ y \\ z \end{pmatrix} = y\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$V^2(2) = \text{Span}\left\{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right\}$$

$$\underset{\sim}{X_2}(t) = e^{2t}\left(\underset{\sim}{V_2} + t(A-2I)\cdot \underset{\sim}{V_2}\right) = e^{2t}\left[\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 & 1 & 1 \\ 2 & -1 & -1 \\ -3 & 2 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right]$$

- $V^3(2)$

$$V^3(2) = \mathbb{R}^3 = \text{Span}\left\{\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\} \qquad \begin{vmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} \neq 0$$ linealki independienteak

$$\underset{\sim}{X_3}(t) = e^{2t}\left[\underset{\sim}{V_3} + t(A-2I)\underset{\sim}{V_3} + \frac{t^2}{2}(A-2I)^2\underset{\sim}{V_3}\right]$$

ADIB

$\to \underset{\sim}{g}(t)$ BEKTORE ASKEA

$$\underset{\sim}{X}' = \underbrace{\begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}}_{P(t)} \cdot \underset{\sim}{X} + \underbrace{\begin{pmatrix} 2e^{-t} \\ 3t \end{pmatrix}}_{\underset{\sim}{g}(t)}$$

① SISTEMA HOMOGENEOA PLANTEATU

$$\underset{\sim}{X}' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \cdot \underset{\sim}{X}$$

$\boxed{1}$ AUTOBALIOAK AURKITU

$$P_A(t) = |A - t I_2| = \begin{vmatrix} -2-t & 1 \\ 1 & -2-t \end{vmatrix} = (-2-t)^2 - 1 = 4 + t^2 - 4t - 1 =$$
$$t^2 - 4t + 3$$

$$\hookrightarrow t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2} \begin{cases} \boxed{t=3} \\ \boxed{t=1} \end{cases}$$

$Spec(A) = \{3, 1\}$ AUTOBALIO SINPLEAK

$\boxed{2}$ AUTOBEKTOREAK KALKULATU

$\to$ Polinomio karakteristikoa $\lambda = 3$ autobalioa ordezkatu

$\boxed{\lambda = 3}$

$$\begin{pmatrix} x \\ y \end{pmatrix} \in V(3) \longleftrightarrow \begin{pmatrix} -5 & 1 \\ 1 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

dim 2
heina 1
dimker = 1 autobektore 1

$$-5x + y = 0 \to \begin{cases} x = x \\ y = 5x \end{cases} \to \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 5x \end{pmatrix} = x \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$\boxed{\underset{\sim}{X}_1 = e^{3t} \begin{pmatrix} 1 \\ 5 \end{pmatrix}}$$

$\boxed{\lambda = 1}$

$$\begin{pmatrix} x \\ y \end{pmatrix} \in V(1) \longleftrightarrow \begin{pmatrix} -3 & 1 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

dim = 2
heina = 1
dimker = 1

$$-3x + y = 0 \to \begin{cases} x = x \\ y = 3x \end{cases} \to \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 3x \end{pmatrix} = x \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\boxed{\underset{\sim}{X}_2 = e^{t} \begin{pmatrix} 1 \\ 3 \end{pmatrix}}$$

FUNTSEZKO MATRIZEA $\varphi(t) = \begin{pmatrix} e^{3t} & e^{t} \\ 5e^{3t} & 3e^{t} \end{pmatrix}$ NON $\varphi(t) = \left(\underset{\sim}{X}_1 \,\middle|\, \underset{\sim}{X}_2 \right)$

② PAM

$$x_p(t) = \varphi(t)\cdot u(t) = \varphi(t)\cdot\begin{pmatrix} u_1(t)\\ u_2(t)\end{pmatrix}$$

$$\varphi(t)\cdot\begin{pmatrix} u_1'(t)\\ u_2'(t)\end{pmatrix} = g(t) \quad \text{SISTEMA LINEALA}$$

$$|\varphi(t)| = \left(e^{3t}\,3e^{t}\right) - \left(5e^{3t}\,e^{t}\right) = 3e^{4t} - 5e^{4t} = -2e^{4t}$$

CRAMMER $u_1'(t)$ ETA $u_2'(t)$ LORTZEKO

$$u_1'(t) = \frac{\begin{vmatrix} 2e^{-t} & e^{t}\\ 3t & 3e^{t}\end{vmatrix}}{-2e^{4t}} = \frac{(2e^{-t}\cdot 3e^{t}) - (3te^{t})}{-2e^{4t}} = \frac{5 - 3te^{t}}{-2e^{4t}}$$

$$u_2'(t) = \frac{\begin{vmatrix} e^{3t} & 2e^{-t}\\ 5e^{3t} & 3t\end{vmatrix}}{-2e^{4t}} = \frac{(e^{3t}\cdot 3t) - (2e^{-t}\cdot 5e^{3t})}{-2e^{4t}} = \frac{3e^{3t}t - 10e^{2t}}{-2e^{4t}}$$

- $$u_1(t) = \int u_1'(t)\,dt = \int\left(-\frac{5}{2e^{4t}}\right)dt + \int\frac{3te^{t}}{2e^{4t}}\,dt = -\frac{5}{2}\left(-\frac{1}{4}\cdot e^{-4t}\right) + \frac{3}{2}$$

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ADIB.

$$\underset{\sim}{X}'(t) = \underbrace{\begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}}_{P(t)} \underset{\sim}{X} + \underbrace{\begin{pmatrix} e^{-2t} \\ -2e^{t} \end{pmatrix}}_{q}$$

1. ORDENEKO SISTEMA LINEAL EZ-HOMOGENEOA

① SISTEMA HOMOGENEOA PLANTEATU

$$\underset{\sim}{X}'(t) = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \underset{\sim}{X}$$

$\boxed{1}$ AUTOBALIOAK AURKITU

$$\longrightarrow P_A(t) = |A - tI_2| = \begin{vmatrix} 1-t & 1 \\ 4 & -2-t \end{vmatrix} = t^2 + t - 6 = (t+3)(t-2)$$

$\mathrm{Spec}\,(A) = \{-3, 2\}$ SINPLEAK (behin bakarrik agertu)

$\boxed{2}$ AUTOBEKTOREAK KALKULATU

$\boxed{\lambda = -3}$

$$\begin{pmatrix} x \\ y \end{pmatrix} \in V(-3) \longleftrightarrow \begin{pmatrix} 4 & 1 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

dim = 2
heina = 1
dimker = 1

$$\{ 4x + y = 0 \longrightarrow \begin{cases} x = x \\ y = -4x \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -4x \end{pmatrix} = x \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

$$\hookrightarrow V(-3) = \mathrm{Span}\left\{\begin{pmatrix} 1 \\ -4 \end{pmatrix}\right\} \longrightarrow \boxed{\underset{\sim}{X}_1 = e^{-3t}\begin{pmatrix} 1 \\ -4 \end{pmatrix}}$$

$\underset{\sim}{X}_1$ AUTOBEKTOREA

$\boxed{\lambda = 2}$

$$\begin{pmatrix} x \\ y \end{pmatrix} \in V(2) \longleftrightarrow \begin{pmatrix} -1 & 1 \\ 4 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

dim = 2
heina = 1
dimker = 1

$$\{ -x + y = 0 \longrightarrow \begin{cases} x = x \\ y = x \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x \end{pmatrix} = x \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\hookrightarrow V(2) = \mathrm{Span}\left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right\} \longrightarrow \boxed{\underset{\sim}{X}_2 = e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

$\underset{\sim}{X}_2$ AUTOBEKTOREA

$$\Psi(t) = \begin{array}{c} \begin{pmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{pmatrix} \\ \underset{\sim}{X}_1 \quad\quad \underset{\sim}{X}_2 \end{array}$$

FUNTSEZKO MATRIZEA

1. ordenako sistema → HOMOGENEOA → $X(t)$
1. ordenako sistema → EZ-HOMOGENEOA → $\Psi(t)$

$$\underset{\sim}{X}_H = c_1 \underset{\sim}{X}_1 + c_2 \underset{\sim}{X}_2$$

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② PAM APLIKATU $\longrightarrow \Psi \cdot \underset{\sim}{U}' = \underset{\sim}{q}$

$$\begin{pmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{pmatrix} \cdot \begin{pmatrix} U_1'(t) \\ U_2'(t) \end{pmatrix} = \begin{pmatrix} e^{-2t} \\ -2e^{t} \end{pmatrix}$$

$$\underset{\sim}{X}_p = U_1 \underset{\sim}{X}_1 + U_2 \underset{\sim}{X}_2$$

↓ CRAMMER APLIKATU

$$|\Psi| = \begin{vmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{vmatrix} = e^{-t} + 4e^{-t} = 5e^{-t}$$

- $$U_1' = \frac{\begin{vmatrix} e^{-2t} & e^{2t} \\ -2e^{t} & e^{2t} \end{vmatrix}}{5e^{-t}} = \frac{1 + 2e^{3t}}{5e^{-t}} = \frac{1}{5}e^{t} + \frac{2}{5}e^{4t}$$

- $$U_2' = \frac{\begin{vmatrix} e^{-3t} & e^{-2t} \\ -4e^{-3t} & -2e^{t} \end{vmatrix}}{5e^{-t}} = \frac{-2e^{-2t} + 4e^{-5t}}{5e^{-t}} = -\frac{2}{5}e^{-t} + \frac{4}{5}e^{-4t}$$

↓ U_1' ETA U_2' INTEGRATU U_1 ETA U_2 LORTZEKO

- $U_1(t) = \frac{e^{t}}{5} + \frac{1}{10}e^{4t}$
- $U_2(t) = \frac{2}{5}e^{-t} - \frac{e^{-4t}}{5}$

$$\underset{\sim}{U} = \begin{pmatrix} U_1(t) \\ U_2(t) \end{pmatrix} = \begin{pmatrix} \frac{e^{t}}{5} + \frac{1}{10}e^{4t} \\ \frac{2}{5}e^{-t} - \frac{e^{-4t}}{5} \end{pmatrix}$$

↓ $\underset{\sim}{U}$ EZAGUTUTA, $\underset{\sim}{X}_p = \Psi \cdot \underset{\sim}{U}$ ADIERAZPENEAN ORDEZKATU

$$\underset{\sim}{X}_p = \Psi \cdot \underset{\sim}{U} = \begin{pmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{pmatrix} \cdot \begin{pmatrix} \frac{e^{t}}{5} + \frac{1}{10}e^{4t} \\ \frac{2}{5}e^{-t} - \frac{e^{-4t}}{5} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}e^{t} \\ -e^{-2t} \end{pmatrix}$$

$$\underset{\sim}{X}_{so}(t) = \Psi(t)\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \underset{\sim}{X}_p = \begin{pmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{2}e^{t} \\ -e^{-2t} \end{pmatrix} \quad c_1, c_2 \in \mathbb{R}$$

→ SOLUZIO OROKORRA

ARIKETA GEHIGARRIAK

1.

$$y^{(iv)} - y'' - 2y' + 2y = f(t)$$

① HOMOGENEA

$$y^{(iv)} - y'' - 2y' + 2y = 0$$

INDIZE ELUAZIOA

$\to r^4 - r^2 - 2r + 2 = 0$

	1	0	-1	-2	2
1		1	1	0	-2
	1	1	0	-2	0
1		1	2	2	
	1	2	2	0	

$\to r^2 + 2r + 2 = 0$

$$r = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2}$$

$r = \frac{-2-2i}{2} = -1 - i$

$r = \frac{-2+2i}{2} = -1 + i$

BIKOIZA $\boxed{r = 1}$ $\begin{cases} y_1 = e^t \\ y_2 = t e^t \end{cases}$ $\boxed{r = -1 \pm i}$ $\begin{cases} y_3 = e^{-t}\cos(t) \\ y_4 = e^{-t}\sin(t) \end{cases}$

$\to \boxed{y_H = C_1 e^t + C_2 t e^t + C_3 e^{-t}\cos(t) + C_4 e^{-t}\sin(t)}$

② KIM

1) $f(t) = t^2 e^t \longrightarrow y_P = (At^2 + Bt + C)\, e^t \cdot t^2$

2) $f(t) = 7 + \cos(t) \longrightarrow y_P = A + B\sin(t) + C\cos(t)$

3) $f(t) = 6t\, e^{-t}\cos t \longrightarrow y_P = (A + Bt)\, e^{-t}\, (C\cos(t) + D\sin(t))\, t$

4) $f(t) = 2e^{-t} + \pi\sin(t) - t\cos(t)$
 ① ② ③

① $y_1 = Ae^{-t}$

② $y_2 = (B\cos(t) + C\sin(t)) \to$ E+ dugu kontuan hartuko, y_3-a kontuan hartzen dudalako jada.

③ $y_3 = (Dt + E)(F\cos(t) + G\sin(t))$

$\to \boxed{y_P = Ae^{-t} + (Dt + E)(F\cos(t) + G\sin(t))}$

(2)

$(\cos^2 x)\, y'' + 2\cos x \cdot \sin x\, y' + (1+\sin^2 x)\, y = \cos^3 x$

$\frac{y_1}{y_2} = x$; $y_1 = y_2 x$

ADIERAZPENA

$y'' + \frac{2\sin x}{\cos x} y' + \frac{(1+\sin^2 x)}{\cos^2 x} y = \cos x$ $\quad y'' + p(t) y' + q(t) y = 0$

$p(t)$

(1) HOMOGENEOA

$W_{y_1,y_2} = K e^{-\int p(t)dt} = K e^{-\int \frac{2\sin x}{\cos x}dx} = K e^{2\ln(\cos x)} = K\cos^2 x$

$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} y_2 x & y_2 \\ y_2' x + y_2 & y_2' \end{vmatrix} = x\cancel{y_2 y_2'} - \cancel{y_2 y_2'}x - y_2^2 = -y_2^2$

2 adierazpenak berdindu

$\to K\cos^2 x = -y_2^2$; $\xrightarrow{K=-1 \text{ balioa}} -\cos^2 x = -y_2^2$; $y_2^2 = \cos^2 x$; $\boxed{y_2 = \cos x}$

$y_1 = y_2 x$ konstantea izanda $\longrightarrow \boxed{y_1 = x\cos x}$

$\to \boxed{y_H = C_1 x\cos x + C_2 \cos x}$

(2) PAM

$y_P = U_1 x\cos x + U_2 \cos x$

$\Phi(x) \cdot \underset{\sim}{U}'(x) = \underset{\sim}{Q}(x)$; $\begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix} \cdot \begin{pmatrix} U_1'(x) \\ U_2'(x) \end{pmatrix} = \begin{pmatrix} 0 \\ q(x) \end{pmatrix}$

$\begin{pmatrix} x\cos(x) & \cos x \\ \cos(x) - \sin(x) x & -\sin x \end{pmatrix} \cdot \begin{pmatrix} U_1'(x) \\ U_2'(x) \end{pmatrix} = \begin{pmatrix} 0 \\ \cos x \end{pmatrix}$

CRAMMER

$|\Phi(x)| = \begin{vmatrix} x\cos(x) & \cos(x) \\ \cos(x) - \sin(x)x & -\sin(x) \end{vmatrix} = -x\cos(x)\sin(x) - (\cos^2(x) - \cos(x)\sin(x)x) =$

$-x\cancel{\cos(x)\sin(x)} - \cos^2 x + x\cancel{\cos(x)\sin(x)} = -\cos^2 x$

$U_1'(x) = \frac{\begin{vmatrix} 0 & \cos x \\ \cos x & -\sin x \end{vmatrix}}{-\cos^2 x} = \frac{-\cos^2 x}{-\cos^2 x} = 1$

$U_2'(x) = \frac{\begin{vmatrix} x\cos x & 0 \\ \cos(x) - \sin(x)x & \cos x \end{vmatrix}}{-\cos^2 x} = \frac{x\cos^2 x}{-\cos^2 x} = -x$

$$U_1(x) = \int U_1'(x)\,dx = \int 1\,dx = x$$

$$U_2(x) = \int U_2'(x)\,dx = \int (-x)\,dx = -\frac{x^2}{2}$$

↳ $$y_p = U_1\, x\cos x + U_2 \cos x = x^2 \cos x - \frac{x^2}{2}\cos x = x^2\cos x$$

↓ HORTAZ

$$\boxed{y_{SO} = \underbrace{C_1\, x\cos x + C_2 \cos x}_{y_H} + \underbrace{x^2 \cos x}_{y_P}}$$

3.

KOEFIZIENTE KONSTANTEKO $\underset{\sim}{X}' = A \cdot \underset{\sim}{X}$

FUNTSEZKO MATRIZEA

$$\Psi(t) = \begin{array}{c} \\ \\ \end{array}\begin{pmatrix} e^{-t} & -2 & 3-t \\ -e^{-t} & 0 & -1 \\ e^{-t} & 0 & 0 \end{pmatrix}$$
$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \underset{\sim}{x_1} & \underset{\sim}{x_2} & \underset{\sim}{x_3} \end{array}$$

A matrizearen autobalioak?

Autobektoreen espazioak?

Sistemaren soluzioak $\longrightarrow \underset{\sim}{x_1}, \underset{\sim}{x_2}, \underset{\sim}{x_3}$

Sistema homogeneoaren soluzioa: $\underset{\sim}{X} = e^{\lambda t}\, \underset{\sim}{V}$

$\boxed{\underset{\sim}{X_1}}$

$$\underset{\sim}{X_1} = \begin{pmatrix} e^{-t} \\ -e^{-t} \\ e^{-t} \end{pmatrix} = e^{-t}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad \left\{ \begin{array}{l} \lambda = -1 \\ \underset{\sim}{V} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \end{array} \right.$$

$\boxed{\underset{\sim}{X_2}}$

$$\underset{\sim}{X_2} = \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix} \quad \left\{ \begin{array}{l} \lambda = 0 \text{ BIKOITZA} \\ \underset{\sim}{V} = \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix} \end{array} \right.$$

$\boxed{\underset{\sim}{X_3}}$

$$\underset{\sim}{X_3} = \begin{pmatrix} 3-t \\ -1 \\ 0 \end{pmatrix}$$

↳ Aljebraikoki kalkulatu

(14)

(14)

A) $x^2 y'' + 2xy' = 0 \qquad y_1 = 1 \qquad y_2 = ?$

$$y'' + \frac{2}{x} y' = 0 \qquad \frac{2}{x} \rightarrow p(x)$$

WRONSKIANA

$$W_{y_1 y_2} = k e^{-\int p(x)dx} = k e^{-\int \frac{2}{x}dx} = k e^{-2\ln|x|} = k e^{\ln|1/x^2|} = \frac{k}{x^2}$$

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_1' y_2 = y_2'$$

2 adierazpenak berdindu

$$y_2' = \frac{k}{x^2}$$

$\boxed{k=1}$

$$y_2' = x^{-2} \longrightarrow y_2 = \int \frac{1}{x^2}dx = \boxed{-\frac{1}{x}}$$

B) $x^2 y'' - x(x+2)y' + (x+2)y = 0 \qquad y_1 = x$

$$y'' - \frac{(x+2)}{x} y' + \frac{(x+2)}{x^2} y = 0 \qquad p(x)$$

$$W_{y_1 y_2} = k e^{-\int p(x)dx} = k e^{\int 1dx + 2\int \frac{1}{x}dx} = k e^x x^2$$

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} x & y_2 \\ 1 & y_2' \end{vmatrix} = x y_2' - y_2$$

$$k e^x x^2 = x y_2' - y_2 \; ; \quad y_2' - \frac{1}{x} y_2 = k e^x x$$

$\boxed{k=1}$

$$y_2' - \frac{1}{x} y_2 = e^x x$$

$$U(x) = e^{\int p(x)dx} = e^{-\int \frac{1}{x}dx} = e^{-\ln|x|} = \frac{1}{x}$$

$$y_2' \frac{1}{x} - \frac{1}{x^2} y_2 = e^x \; ; \quad \left(y_2 \frac{1}{x}\right)' = e^x$$

INTEGRATU

$$y_2 \frac{1}{x} = e^x \; ; \quad \boxed{y_2 = e^x x}$$

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10.

$$\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \underbrace{\begin{pmatrix} 1/t & 0 \\ -1/t^2 & 1/t \end{pmatrix}}_{P(t)} \cdot \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} + \begin{pmatrix} t^2 \\ 0 \end{pmatrix}$$

$$\underset{\sim}{x}' = P(t)\underset{\sim}{x} + \underset{\sim}{q}(t)$$

$$\begin{pmatrix} x(1) \\ y(1) \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \longrightarrow \text{HASTAPEN BALDINTZA}$$

↓ SISTEMA ERAN

$$\begin{cases} x' = \dfrac{1}{t}x + t^2 & \text{(LEHEN ORDENAKO EDA LINEALA)} \\ y' = -\dfrac{1}{t^2}x + \dfrac{1}{t}y + 0 \end{cases}$$

$\boxed{x}$

x(t) lortu

hastapen baldintzekin ↓ x LORTU

x LORTZEAN y'-ren adierazpenean ordezkatu ↓ y LORTU

$$x' - \frac{1}{t}x = t^2$$

① HOMOGENEOA PLANTEATU

$$x' - \frac{1}{t}x = 0; \quad x' = \frac{1}{t}x; \quad \frac{dx}{dt} = \frac{1}{t}x; \quad \frac{1}{x}dx = \frac{1}{t}dt;$$

$$\ln(x) = \ln(t) + K; \quad x = e^{\ln(t)+K}; \quad \boxed{x_H = tK}$$

② x_p PLANTEATU

$$x_p = tK(t) \qquad \text{ETA} \qquad x'_p = K(t) + tK'(t)$$

↓ EDA OSOAN ORDEZKATU

$$\cancel{K(t)} + tK'(t) - \frac{1}{\cancel{t}}\cancel{t}\,\cancel{K(t)} = t^2; \quad tK'(t) = t^2; \quad K'(t) = t$$

↓ INTEGRATUZ

$$K(t) = \int K'(t)\,dt = \int t\,dt = \frac{t^2}{2}$$

↓ HORTAZ

$$x_p = t\,\frac{t^2}{2} = \frac{t^3}{2} \longrightarrow x_{SO} = tK + \frac{t^3}{2}$$

↓ y'-ren adierazpenean ordezkatu

$$y' = -\frac{1}{t^2}\left(tK + \frac{t^3}{2}\right) + \frac{1}{t}y; \quad y' = -\frac{1}{t}K - \frac{t}{2} + \frac{1}{t}y$$

$$y' - \frac{1}{t}y = -\frac{1}{t} - \frac{t^2}{2}$$

① HOMOGENEOA

$y' - \frac{1}{t}y = 0$; $\frac{dy}{dt} = \frac{1}{t}y$; $\int \frac{1}{y}dy = \int \frac{1}{t}dt$; $\ln|y| = \ln|t| + K$;

$\boxed{y_h = Kt}$

② y_p PLANTEATU

$y_p = K(t)\,t$ ETA $y_p' = K'(t)\,t + K(t)$

↓ EDA OSOAN ORDEZKATU

$K'(t)t + \cancel{K(t)} - \cancel{\frac{1}{t}K(t)t} = -\frac{1}{t}K - \frac{t}{2}$; $K'(t)t = -\frac{1}{t}K - \frac{t}{2}$;

$K'(t) = -\frac{1}{t^2}K - \frac{1}{2} \xrightarrow{\text{INTEGRAN}} K(t) = \int K'(t)dt = -K\int \frac{1}{t^2}dt - \frac{1}{2}\int dt$;

$K(t) = \frac{K}{t} - \frac{1}{2}t$

↓ HORTAZ

$y_p = \left(\frac{K}{t} - \frac{1}{2}t\right)t = K - \frac{1}{2}t^2$

$\hookrightarrow y_{SO} = Kt + K - \frac{1}{2}t^2$

HASTAPEN BALDINTZAK APLIKATUZ

$x(1) = 1 \cdot K + \frac{1^3}{2} = 1$; $K = 1 - 1/2$; $\boxed{K_1 = 1/2}$

$y(1) = 1 \cdot K + K - \frac{1}{2} = -1$; $2K = -1/2$; $\boxed{K_2 = -1/4}$

$$\begin{cases} x(t) = \frac{t}{2} + \frac{t^3}{2} \\ y(t) = -\frac{t}{4} - \frac{1}{4} - \frac{1}{2}t^2 \end{cases}$$

11.

$$\underset{\sim}{x}' = \begin{pmatrix} a & d & e \\ 0 & b & f \\ 0 & 0 & c \end{pmatrix} \cdot \underset{\sim}{x} + \begin{pmatrix} 0 \\ te^{2t} \\ 0 \end{pmatrix}$$

A) a, b, c, d, e, f ?

① $r = 2$ autobalio hirukoitza

↳ A matrizea triangelatuta dagoenez, a, b eta c autobalioak dira.

② $V(2) = \{(\ )\} = \dim(V(2)) = 1 \longrightarrow$ 2 autobalioari autobektore bat

③ $\underset{\sim}{x} = e^{2t}\begin{pmatrix} 6t + \frac{5t^2}{2!} \\ 5t \\ 1 \end{pmatrix}$ Sistema homogeneoaren soluzioetako bat

DERIBATUTA

$$\underset{\sim}{x}'(t) = \begin{pmatrix} [(6+5t) + 2(6t + \frac{5t^2}{2})]e^{2t} \\ (10t+5)e^{2t} \\ 2e^{2t} \end{pmatrix} = \begin{pmatrix} 2 & d & e \\ 0 & 2 & f \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 6t + \frac{5t^2}{2} \\ 5t \\ 1 \end{pmatrix} e^{2t} =$$

$$= e^{2t}\begin{pmatrix} 12t + 5t^2 + 5dt + e \\ 10t + f \\ 2 \end{pmatrix}$$

$$\begin{cases} e = 6 \\ d = 1 \\ f = 5 \end{cases}$$

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B) $\underset{\sim}{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \underset{\sim}{x} + \begin{pmatrix} e^{-2t} \\ -2e^{t} \end{pmatrix}$

① HOMOGENEOA

Ⓐ AUTOBALIOAK

$P_A = \begin{vmatrix} 1-t & 1 \\ 4 & -2-t \end{vmatrix} = [(1-t)(-2-t)] - 4 = -2 - t + 2t + t^2 - 4 = t^2 + t - 6$

$t = \dfrac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1} = \dfrac{-1 \pm 5}{2}$ → $\boxed{t = 2}$, $\boxed{t = -3}$

$Spec(A) = \{2, -3\}$

Ⓑ AUTOBEKTOREAK

dim = 2
homo = 1
dimker = 1

• $\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix} \in V(2) \longleftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 4 & -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-x + y = 0 \longrightarrow \begin{cases} x = x \\ y = x \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x \end{pmatrix} = x \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\boxed{\underset{\sim}{\chi}_1 = e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}}$

• $\underset{\sim}{v} = \begin{pmatrix} x \\ y \end{pmatrix} \in V(-3) \longleftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$4x + y = 0 \longrightarrow \begin{cases} x = x \\ y = -4x \end{cases} \longrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -4x \end{pmatrix} = x \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

$\boxed{\underset{\sim}{\chi}_2 = e^{-3t} \begin{pmatrix} 1 \\ -4 \end{pmatrix}}$

$y_H = C_1 e^{2t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 e^{-3t} \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

② PAM

FUNTSEZKO MATRIZEA $\to \Psi(t) = \left(\underset{\sim}{\chi}_1 \mid \underset{\sim}{\chi}_2 \right)$

$y_P = u_1 \underset{\sim}{\chi}_1 + u_2 \underset{\sim}{\chi}_2$

FUNTSEZKO MATRIZEA $\leftarrow \Psi(t) = \begin{pmatrix} e^{2t} & e^{-3t} \\ e^{2t} & -4e^{-3t} \end{pmatrix}$

$\Psi(t) \cdot u'(t) = g(t) \ ; \ \begin{pmatrix} e^{2t} & e^{-3t} \\ e^{2t} & -4e^{-3t} \end{pmatrix} \begin{pmatrix} u_1'(t) \\ u_2'(t) \end{pmatrix} = \begin{pmatrix} e^{-2t} \\ -2e^{-t} \end{pmatrix}$

CRAMMER

$$|\Psi(t)| = \begin{vmatrix} e^{2t} & e^{-3t} \\ e^{2t} & -4e^{-3t} \end{vmatrix} = -4e^{2t}e^{-3t} - e^{-3t}e^{2t} = -4e^{-t} - e^{-t} = -5e^{-t}$$

$$U_1'(t) = \frac{\begin{vmatrix} e^{-2t} & e^{-3t} \\ -2e^{-t} & -4e^{-3t} \end{vmatrix}}{-5e^{-t}} = \frac{-4e^{-2t}e^{-3t} + 2e^{-t}e^{-3t}}{-5e^{-t}} = \frac{-4e^{-5t} + 2e^{-4t}}{-5e^{-t}} = \frac{4}{5}e^{-4t} - \frac{2}{5}e^{-3t}$$

$$U_2'(t) = \frac{\begin{vmatrix} e^{2t} & e^{-2t} \\ e^{2t} & -2e^{-t} \end{vmatrix}}{-5e^{-t}} = \frac{-2e^{2t}e^{-t} - e^{2t}e^{-2t}}{-5e^{-t}} = \frac{-2e^{t} - 1}{-5e^{-t}} = \frac{2}{5}e^{2t} + \frac{1}{5}e^{t}$$

$\downarrow$ INTEGRATU $U_1(t)$ ETA $U_2(t)$ LORTZEKO

$$U_1(t) = \int U_1'(t)\,dt = \frac{4}{5}\int e^{-4t}\,dt - \frac{2}{5}\int e^{-3t}\,dt = -\frac{1}{5}e^{-4t} + \frac{2}{15}e^{-3t}$$

$$U_2(t) = \int U_2'(t)\,dt = \frac{2}{5}\int e^{2t}\,dt + \frac{1}{5}\int e^{t}\,dt = \frac{1}{5}e^{2t} + \frac{1}{5}e^{t}$$

$\downarrow$

$$\underset{\sim}{X_p}(t) = \Psi(t)\cdot\begin{pmatrix} U_1(t) \\ U_2(t) \end{pmatrix} = \overset{2\times 2}{\begin{pmatrix} e^{2t} & e^{-3t} \\ e^{2t} & -4e^{-3t} \end{pmatrix}} \overset{2\times 1}{\begin{pmatrix} -\frac{1}{5}e^{-4t} + \frac{2}{15}e^{-3t} \\ \frac{1}{5}e^{2t} + \frac{1}{5}e^{t} \end{pmatrix}} =$$

$$= \begin{pmatrix} \cancel{-\frac{1}{5}e^{-2t}} + \frac{2}{15}e^{-t} + \frac{1}{5}e^{-t} \cancel{+ \frac{1}{5}e^{-2t}} \\ -\frac{1}{5}e^{-2t} + \frac{2}{15}e^{-t} - \frac{4}{5}e^{-t} - \frac{4}{5}e^{-2t} \end{pmatrix} = \begin{pmatrix} \frac{1}{3}e^{-t} \\ -e^{-2t} - \frac{2}{3}e^{-t} \end{pmatrix}$$

13.

$$y'' + 2x^{-1} y' + e^{x} y = 0$$

$W_{y_1 y_2} = 2 \quad x = 1$ puntuan

$W_{y_1 y_2}\ ? \quad x = 5$ denean

$$W_{y_1 y_2} = K e^{-\int p(x) dx} = K e^{-\int 2 \frac{1}{x} dx} = K e^{-2 \ln|x|} = K e^{\ln|1/x^2|} = \frac{K}{x^2}$$

$$\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_1' y_2 = 2$$

$$W_{y_1 y_2} = 2 = K \frac{1}{x^2} \xrightarrow{x=1} \boxed{K = 2}$$

$x = 5$ denean $\longrightarrow W_{y_1 y_2} = \frac{2}{5^2} = \frac{2}{25}$

16.

$$4x^2 y'' - (12x^2 + 4x) y' + (6x + 3) y = 0 \qquad y = x^{a}$$

$$y'' - \frac{12x^2 + 4x}{4x^2} y' + \frac{6x + 3}{4x^2} y = 0 \longrightarrow y'' - \left(3 + \frac{1}{x}\right) y' + \left(\frac{3}{2x} + \frac{3}{4x^2}\right) y = 0$$

$$W_{y_1 y_2} = K e^{-\int p(x) dx} = K e^{+\int (3 + \frac{1}{x}) dx} = K e^{3x + \ln|x|} = K e^{3x} x$$

$$\frac{y_2}{y_1} = e^{3x}$$

$$W_{y_1 y_2} = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_1' y_2$$

$$(e^{3x})' = \left(\frac{y_2}{y_1}\right)' = 3e^{3x} = \frac{y_2' y_1 - y_2 y_1'}{y_1^2} = \frac{W_{y_1 y_2}}{y_1^2}; \quad W_{y_1 y_2} = 3 y_1^2 e^{3x}$$

$$K e^{3x} x = 3 y_1^2 e^{3x}; \quad y_1^2 = x; \quad \boxed{y_1 = \sqrt{x}}$$

$$y_2 = e^{3x} y_1; \quad \boxed{y_2 = e^{3x} \sqrt{x}}$$

$$\boxed{y_{SO} = C_1 \sqrt{x} + C_2 e^{3x} \sqrt{x}}$$

15

15.

$$x^2y'' - x(x+2)y' + (x+2)y = 0 \xrightarrow[\text{kendu}]{y''\text{-ren aurrean dagoena}} y'' - \frac{(x+2)}{x}y' + \frac{(x+2)}{x^2}y = 0$$

$\frac{(x+2)}{x} \rightarrow P$

Badakigu 2 soluzioen arteko zatidura e^x dela $\longrightarrow e^x = \frac{y_2}{y_1}$

WRONSKIARRA $\longrightarrow W_{y_1,y_2} = \left|\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}\right| = y_1y_2' - y_2y_1'$

$$(e^x)' = \left(\frac{y_2}{y_1}\right)'; \quad e^x = \frac{y_1y_2' - y_2y_1'}{y_1^2} = \frac{W_{y_1,y_2}}{y_1^2}; \quad W_{y_1,y_2} = e^x y_1^2$$

ABELEN FORMULA APLIKATUZ

$$\int \frac{x}{x}dx + 2\int \frac{1}{x}dx = x + \ln(x^2)$$

$$W_{y_1,y_2} = \left[e^x y_1^2 = k\, e^{\int A_{zt}(P)} = k\, e^{\int\left(\frac{x+2}{x}\right)dx} = k\, e^x x^2\right]$$

HORTAZ

$$e^x y_1^2 = k e^x x^2; \quad y_1^2 = kx^2; \quad \boxed{y_1 = kx} \xrightarrow[\text{IZANIK}]{e^x = \frac{y_2}{y_1}} \frac{y_2}{e^x} = kx;$$

$$\boxed{y_2 = ke^x x}$$

$$y_{so}(x) = C_1y_1 + C_2y_2$$

$$\longrightarrow \boxed{y_{so}(x) = C_1x + C_2e^x x}$$

ADIB.

$(2x+1)y'' + (4x-2)y' - 8y = 0$ $\quad y = e^{mx}$ soluzio bat dela jakinik

1) AURKITU m

$$\rightarrow y'' + \frac{4x-2}{2x+1}y' - \frac{8}{2x+1}y = 0$$

$$y = e^{mx} \longrightarrow y' = me^{mx} \longrightarrow y'' = m^2e^{mx}$$

HASIERAKO ADIERAZPENEAN ORDEZKATU

$$(2x+1)m^2e^{mx} + (4x-2)me^{mx} - 8e^{mx} = 0;$$

$$e^{mx}\left[(2x+1)m^2 + (4x-2)m - 8\right] = 0; \quad x\underbrace{(2m^2+4m)}_{①} + \underbrace{(m^2-2m-8)}_{②} = 0 \quad \forall x$$

$$\begin{cases} ① & 2m^2+4m = 0 \\ ② & m^2-2m-8 = 0 \end{cases} \longrightarrow \begin{cases} m=0, & m=-2 \\ m=-2, & m=4 \end{cases} \longrightarrow \boxed{m=-2}$$

HORTAZ

$$\boxed{y = e^{-2x}}$$

SKIARRA $\longrightarrow W_{y_1,y_2} = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-2x} & y_2 \\ -2e^{-2x} & y_2' \end{vmatrix} = e^{-2x}y_2' + 2e^{-2x}y_2 = e^{-2x}(y_2' + 2y_2)$

(11)

$$Ke^{\int A_{21}(p)} = Ke^{\int -\frac{4x+2}{2x+1}dx} = \ldots = K(1+2x)^2 \cdot e^{-2x}$$

↓ WRONSKIARRAREN 2 ADIERAZPENAK BERDINDU

$$\cancel{e^{-2x}}(y_2' + 2y_2) = K(1+2x)^2 \cancel{e^{-2x}} \;;\; y_2' + 2y_2 = K(1+2x)^2$$

1) y_H AURKITU (HOMOGENEOA)

$$y_2' + 2y_2 = 0 \;;\; y_2' = -2y_2 \;;\; \frac{dy}{dx} = -2y_2 \;;\; \int \frac{1}{y_2}dy = -2\int dx;$$

$$\ln y_2 = -2x + K; \; \boxed{y_2 = e^{-2x}K} = y_H$$

2) $y_P = e^{2x}K(x)$ ETA $y_P' = -2e^{-2x}K(x) + e^{2x}K'(x)$

↓ EDA OSOAN ORDEZKATU

$$\cancel{2e^{-2x}K(x)} + e^{2x}K'(x) - \cancel{2e^{-2x}K(x)} = K(1+2x)^2;$$

$$K'(x) = K\frac{(1+2x)^2}{e^{-2x}} \xrightarrow{\text{INTEGRATU}} K(x) = \frac{(1+4x^2)}{e^{-2x}}$$

$$y_2 = K_1 e^{-2x} + K_2(1+4x^2)$$

$\hookrightarrow y_{SO} = C_1 e^{-2x} + C_2(1+4x^2) \qquad C_1, C_2 \in \mathbb{R}$

AD IB

$a_0 y'' + \ldots$

$P_A(t) \ldots$

OROKO...

$P_A(t)$

$P_A(r)$

$V = \ldots$

(A...

X

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ADIB.

$y''' + y' = \cos t$ — 3. ordeneko EDA lineal ez-homogeneoa

↳ 3. ordeneko KEKEDALEH

① HOMOGENEOA

$y''' + y' = 0$

$r^3 + r = 0 \longrightarrow Spec(A) = \{0, i, -i\}$ SINPLEAK

$$\boxed{y_H} = \underbrace{C_1}_{y_1} + C_2 \underbrace{\cos t}_{y_2} + \boxed{C_3 \underbrace{\sin t}_{y_3}} = y_H$$

② PAM

$$\varphi(t) = \begin{pmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{pmatrix}$$

- $\underset{\sim}{x}' = P(t) \cdot \underset{\sim}{x} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ \cos t \end{pmatrix}}_{\underset{\sim}{q}(t)}$

FUNTSEZKO MATRIZEA $\longrightarrow \varphi(t) = \begin{pmatrix} 1 & \cos t & \sin t \\ 0 & -\sin t & \cos t \\ 0 & -\cos t & -\sin t \end{pmatrix}$ $\quad |\varphi| = 1$

$\underset{\sim}{x_p} = \varphi \cdot \underset{\sim}{x} \longrightarrow \varphi \cdot \underset{\sim}{u}' = \underset{\sim}{q}$

$\varphi \cdot \begin{pmatrix} u_1' \\ u_2' \\ u_3' \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \cos t \end{pmatrix}$ $\quad x_p = u_1 + u_2 \cos t + u_3 \sin t$

↓ CRAMMER EGINEZ EBATZI

$$\begin{cases} u_1' = \cos t \\ u_2' = -\cos^2 t \\ u_3' = -\cos t \sin t \end{cases} \xrightarrow{\text{INTEGRATUZ}} \begin{cases} u_1 = \sin t \\ u_2 = -\dfrac{t + \cos t \sin t}{2} \\ u_3 = \cos^2 t \end{cases}$$

$x_1 = y \longrightarrow x_1' = y' = x_2$

$x_2 = y' \qquad x_2' = y'' = x_3$

$y_3 = y'' \qquad x_3' = y''' = \cos t - y' = \cos t - x_2$

$$P(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\underset{\sim}{x}_p(t) = \Psi(t)\cdot \underset{\sim}{u}(t) = \begin{pmatrix} 1 & \cos t & \sin t \\ 0 & -\sin t & \cos t \\ 0 & -\cos t & -\sin t \end{pmatrix} \cdot \begin{pmatrix} \sin t \\ -\dfrac{t + \cos t \sin t}{2} \\ \cos^2 t \end{pmatrix} =$$

$$= \begin{pmatrix} \sin t - \dfrac{t\cos t}{2} - \cancel{\dfrac{\cos^2 t \sin t}{2}} + \cancel{\dfrac{\sin t \cos^2 t}{2}} \\ \text{EZ DA BEHARREZKOA} \\ \\ \end{pmatrix} = \begin{pmatrix} y_p \\ y_p' \\ y_p'' \end{pmatrix}$$

$$\boxed{y_{SO}(t) = C_1 + C_2 \cos t + C_3 \sin t + \underbrace{\sin t - \frac{t\cos t}{2}}_{y_p}}$$

ADIB.

$y''' + y' = \cos t \longrightarrow \begin{cases} L[y] \text{ KEKEDALH} \\ g(t) = \cos t \end{cases}$ $\boxed{\text{KIM}}$

INDIZE EKUAZIOA $\longrightarrow r^3 + r = 0;\ r(r^2+1)=0 \rightarrow \{0, i, -i\}$

$\hookrightarrow y_{so} = C_1 + C_2 \cos t + C_3 \sin t$

$\downarrow$ HORTAZ $\boxed{\text{KIM}}$ APLIKATUZ

$y_p(t) = (A\cdot \cos t + B \sin t)\, x^{s} = A t \cos t + B t \sin t$

$\boxed{s=0}$ $y_p = A \cos t + B \sin t$ EZ EKUAZIO HOMOGENEOAREN SOLUZIOA DELAKO

$\boxed{s=1}$ $y_p = A t \cos t + B t \sin t$ $\boxed{\text{BAI}}$

$y'_p = (A+Bt)\cos t + (B - At)\sin t$

$y''_p = \cdots$

$y'''_p = -(3A + Bt)\cos t + (-3B + At)\sin t$

$\downarrow$ HASIERAKO EKUAZIOAN ORDEZKATUZ

$\cos t\,(A + Bt - 3A - Bt) + \sin t\,(B - At - 3B + At) = \cos t \quad \forall t$

$\cos t\,\underbrace{(-2A)}_{1} + \sin t\,\underbrace{(-2B)}_{0} = \cos t \quad \forall t$

$-2A = 1;\ \boxed{A = -1/2}$

$-2B = 0;\ \boxed{B = 0}$ $\longrightarrow \boxed{y_{so} = C_1 + C_2 \cos t + C_3 \sin t - \frac{1}{2} t \cos t}$

y_p

ADIB.

$y'' + y' - 2y = x^3 e^{4x}$

① HOMOGENEOA

$y'' + y' - 2y = 0$

INDIZE EKUAZIOA $\to r^2 + r - 2 = 0$; $r = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-2)}}{2} = \frac{-1 \pm 3}{2}$ $\boxed{r = -2}$ $\boxed{r = 1}$

$\begin{cases} y_1 = e^{-2x} \\ y_2 = e^x \end{cases} \longrightarrow y_H = C_1 e^{-2x} + C_2 e^x$

② KIM

$q = x^3 e^{4x}$

$y_1 = (Ax^3 + Bx^2 + Cx + D) e^{4x}$

$y_1' = (3Ax^2 + Bx + C)e^{4x} + 4(Ax^3 + Bx^2 + Cx + D)e^{4x}$

$y_1'' = (6Ax + B)e^{4x} + (3Ax^2 + Bx + C)e^{4x} + 4(3Ax^2 + Bx + C)e^{4x} + 16(Ax^3 + Bx^2 + Cx + D)e^{4x}$

$y_1'' = 5(3Ax^2 + Bx + C)e^{4x} + 16(Ax^3 + Bx^2 + Cx + D)e^{4x} + (6Ax + B)e^{4x}$

Honen ostean y_1-en adierazpenak hasierako ekuazioan ordezkatu konstanteen balioak lortzeko.

$y_{SO} = y_H + y_P = C_1 e^{-2x} + C_2 e^x + y_P$

ADIB.

$y^{(IV)} + 4y'' = \underbrace{\sin(2t)}_{q_1} + \underbrace{te^t}_{q_2} + \underbrace{4}_{q_3}$

① HOMOGENEOA

$y^{(IV)} + 4y'' = 0$

INDIZE EKUAZIOA $\to r^4 + 4r^2 = 0$; $r^2(r^2 + 4) = 0$ $\boxed{r = 0}$ BIKOITZA $\boxed{r = \sqrt{-4} = \pm 2i}$

$\boxed{r = 0}$ $\begin{cases} y_1 = e^{0t} = 1 \\ y_2 = t \end{cases}$ $\boxed{r = \pm 2i}$ $\begin{cases} y_3 = \cos 2t \\ y_4 = \sin 2t \end{cases}$

$\boxed{y_H = C_1 + C_2 t + C_3 \cos 2t + C_4 \sin 2t}$

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ADIB.

$$y'' + y' - 2y = \underbrace{(x+1)e^x}_{g_1} + \underbrace{x^2 e^{3x}\sin(7x)}_{g_2}$$

① HOMOGENEOA

$y'' + y' - 2y = 0$

$\hookrightarrow \boxed{y_H = C_1 e^x + C_2 e^{-2x}}$

② KIM

$$y_P = (Ax+B)e^x x + e^{3x}\left[(Cx^2+Dx+E)\sin(7x) + (Gx^2+Hx+I)\cos(7x)\right]$$

AURREKO ADIBIDEAREN JARRAIPENA

② KIM

(1) $g_1 = \sin(2t)$

$\downarrow$

$y_1 = (B\cos 2t + C\sin 2t)\,t$

(2) $g_2 = te^t$

$\downarrow$

$y_2 = (Dt+E)e^t$

(3) $g_3 = 4$

$\downarrow$

$y_3 = Ft^2$

$\hookrightarrow y_P = (B\cos(2t) + C\sin(2t))t + (Dt+E)e^t + Ft^2$

$$\hookrightarrow \boxed{y_{Oc} = \underbrace{C_1 + C_2 t + \sin(2t)C_3 + \cos(2t)C_4}_{y_H} + \underbrace{(B\cos(2t) + C\sin(2t))t + (Dt+E)e^t + Ft^2}_{y_P}}$$

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A) $y''' + y'' + y' + y = \underbrace{e^{-x}}_{g_1} \underbrace{+4x}_{g_2}$ $\boxed{\text{KIM}}$

① HOMOGENEDA

$r^3 + r^2 + r + 1 = 0$

$$\begin{array}{c|cccc} & 1 & 1 & 1 & 1 \\ -1 & & -1 & 0 & -1 \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$$

$\hookrightarrow r^2 + 1 = 0$; $\boxed{r = \pm i}$

$y_1 = \cos(x)$ $\qquad y_3 = e^{-x}$

$y_2 = \sin(x)$

$y_H = C_1 \cos(x) + C_2 \sin(x) + C_3 e^{-x}$

② KIM

- $g_1 = e^{-x}$

$y_1 = Ae^{-x} x \longrightarrow \boxed{y = \frac{1}{2} e^{-x} x}$

$y_1' = -Ae^{-x} x + Ae^{-x}$

$y_1'' = Ae^{-x} x - Ae^{-x} - Ae^{-x}$

$y_1''' = -Ae^{-x} x + Ae^{-x} + Ae^{-x} + Ae^{-x}$

$\downarrow$

$\cancel{-Ae^{-x}x} + Ae^{-x} + Ae^{-x} + Ae^{-x} \cancel{+Ae^{-x}x} - Ae^{-x} - Ae^{-x} \cancel{-Ae^{-x}x} + Ae^{-x} \cancel{+Ae^{-x}x} = e^{-x}$;

$2Ae^{-x} = e^{-x}$; $\boxed{A = 1/2}$

- $g_2 = 4x$

$y_2 = (Bx + C) \longrightarrow y = 4x - 4$

$y_2' = B$

$y_2'' = 0$

$y_2''' = 0$

$\downarrow$

$B + Bx + C = 4x$; $\begin{cases} B + C = 0 \ ; \ \boxed{C = -4} \\ \boxed{B = 4} \end{cases}$

$y_{SG}(x) = C_1 \cos(x) + C_2 \sin(x) + C_3 e^{-x} + \frac{1}{2} e^{-x} x + 4x - 4$

ADIB

$$y'' + y = \frac{1}{\cos x}$$

① HOMOGENEOA PLANTEATU

INDIZE EKUAZIOA $\hookrightarrow r^2 + 1 = 0$; $\boxed{r = \pm i}$ $\longrightarrow$ KONPLEXUAK

$$\begin{cases} y_1 = \cos(x) = e^{at}\cos bt = e^{0t}\cos t \\ y_2 = \sin(x) = e^{at}\sin bt = e^{0t}\sin t \end{cases}$$

$$\boxed{y_{50} = C_1\cos(x) + C_2\sin(x)}$$

② PAM

$$y_P = U_1\cos(x) + U_2\sin(x)$$

...REKO ...ERA $\leftarrow$ $\varphi(x) \cdot \underset{\sim}{U'}(x) = \underset{\sim}{g}(x)$; $\begin{pmatrix} y_1 & y_2 \\ y_1' & y_2' \end{pmatrix} \cdot \begin{pmatrix} U_1'(x) \\ U_2'(x) \end{pmatrix} = \begin{pmatrix} 0 \\ g(x) \end{pmatrix}$

$$\begin{pmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{pmatrix} \cdot \begin{pmatrix} U_1'(x) \\ U_2'(x) \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\cos(x)} \end{pmatrix}$$

$\downarrow$ CRAMMER APLIKATUZ EBATZI

$$|\varphi(x)| = \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix} = \cos^2(x) + \sin^2(x) = 1$$

$$U_1'(x) = \begin{vmatrix} 0 & \sin(x) \\ \frac{1}{\cos(x)} & \cos(x) \end{vmatrix} = -\frac{\sin(x)}{\cos(x)}$$

$$U_2'(x) = \begin{vmatrix} \cos(x) & 0 \\ -\sin(x) & \frac{1}{\cos(x)} \end{vmatrix} = 1$$

$$U_1(x) = \int U_1'(x)\,dx = \int \left(-\frac{\sin(x)}{\cos(x)}\right) dx = \ln|\cos(x)|$$

$$U_2(x) = \int 1\,dx = x$$

$$\boxed{y_{50} = \underbrace{C_1\cos(x) + C_2\sin(x)}_{y_H} + \underbrace{\ln|\cos(x)|\cdot\cos(x) + x\sin(x)}_{y_P}}$$

ADIB.

$$y'' + y' - 2y = x^2 + 5x + 1 \longrightarrow \text{2 mailako polinomioa}$$

① HOMOGENEOA

$$y'' + y' - 2y = 0$$

INDIZE EKUAZIOA

$$\rightarrow r^2 + r - 2 = 0 \; ; \quad r = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{-1 \pm 3}{2} \quad \boxed{r = -2} \quad \boxed{r = 1}$$

$$y_1 = e^{-2x}$$

$$y_2 = e^{x}$$

$$\longrightarrow \boxed{y_H = C_1 e^{-2x} + C_2 e^{x}}$$

② KIM

$$f = x^2 + 5x + 1$$

$$y_1 = Ax^2 + Bx + C$$

$$y_1' = 2Ax + B$$

$$y_1'' = 2A$$

↓ HASIERAKO ADIERAZPENEAN ORDEZKATU

$$y'' + y' - 2y = x^2 + 5x + 1 \; ; \quad 2A + 2Ax + B + -2Ax^2 - 2Bx - 2C = x^2 + 5x + 1 \; ;$$

$$x^2(-2A) + x(2A - 2B) + 2A + B - 2C = x^2 + 5x + 1$$

$$x^2: \quad -2A = 1 \; ; \quad \boxed{A = -1/2}$$

$$x: \quad 2A - 2B = 5 \; ; \quad -1 - 2B = 5 \; ; \quad \boxed{B = -3}$$

$$2A + B - 2C = 1 \; ; \quad -1 - 3 - 2C = 1 \; ; \quad \boxed{C = -5/2}$$

$$\boxed{y_P = -\frac{x^2}{2} - 3x - \frac{5}{2}}$$

$$\boxed{y_{SO} = \underbrace{C_1 e^{-2x} + C_2 e^{x}}_{y_H} \underbrace{- \frac{x^2}{2} - 3x - \frac{5}{2}}_{y_P}}$$

$(Ax^2 + Bx + C)\, x^S$ establitzerakoan, lehenengo adierazpena y_1 edo y_2-ren berdina irteten bada, S-ren balioa aldatuko dugu hau gertatzen ez den arte

(M)

19.

A) $y'' + y' + y = \sin^2 x = \underbrace{\frac{1}{2}}_{g_1} - \underbrace{\frac{\cos 2x}{2}}_{g_2}$

① HOMOGENEOA

$y'' + y' + y = 0$

INDITE EKUAZIOA $\to r^2 + r + 1 = 0$; $r = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$

$\to \boxed{r = \frac{-1 + \sqrt{3}i}{2}}$

ERRO KONPLEXUAK

$\to \boxed{r = \frac{-1 - \sqrt{3}i}{2}}$

$y_H = C_1 e^{-x/2} \cos(\sqrt{3}/2\, x) + C_2 e^{-x/2} \sin(\sqrt{3}/2\, x)$

② $\boxed{\text{KIM}}$

$\boxed{g_1} = \frac{1}{2} \to$ $y_1 = A$, $y_1' = 0$, $y_1'' = 0$ $\quad \boxed{A = \frac{1}{2}}$ $\to y_1 = \frac{1}{2}$

$y_1 = A$

$\boxed{g_2} = \frac{\cos(2x)}{2}$

$y_2 = (B\cos(2x) + C\sin(2x))$

$\to y_2 = B\cos(2x) + C\sin(2x)$

$y_2' = -2B\sin(2x) + 2C\cos(2x)$

$y_2'' = -4B\cos(2x) - 4C\sin(2x)$

$\to -4B\cos(2x) - 4C\sin(2x) - 2B\sin(2x) + 2C\cos(2x) + B\cos(2x) + C\sin(2x) = \frac{\cos 2x}{2}$

$-3B\cos(2x) + 2C\cos(2x) - 3C\sin(2x) - 2B\sin(2x) = \frac{\cos 2x}{2}$;

$\cos(2x) \cdot (-3B + 2C) + \sin(2x) \cdot (-3C - 2B) = \frac{\cos 2x}{2}$

$-3B + 2C = \frac{1}{2}$; $\frac{9}{2}C + 2C = \frac{1}{2}$; $\frac{13}{2}C = \frac{1}{2}$; $\boxed{C = 1/13}$

$-3C - 2B = 0$; $\underline{B = -3/2\, C}$; $B = -\frac{3}{2} \cdot \frac{1}{13}$; $\boxed{B = -\frac{3}{26}}$

$\to y_2 = -\frac{3}{26}\cos(2x) + \frac{1}{13}\sin(2x)$

HORTAZ

$y_{SO} = C_1 e^{-x/2} \cos(\sqrt{3}/2\, x) + C_2 e^{-x/2} \sin(\sqrt{3}/2\, x) + \frac{1}{2} + \frac{3}{26}\cos(2x) - \frac{1}{13}\sin(2x)$

B) $y'' + y' + 4y = 2\sinh x = 2\dfrac{e^x - e^{-x}}{2} = 2\left(\dfrac{e^x}{2} - \dfrac{e^{-x}}{2}\right) = e^x - e^{-x}$

① HOMOGENEOA

$r^2 + r + 4 = 0$; $r = \dfrac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \dfrac{-1 \pm \sqrt{-15}}{2} = \dfrac{-1 \pm \sqrt{15}i}{2}$

$\rightarrow \boxed{r = \dfrac{-1 + \sqrt{15}i}{2}}$

$\rightarrow \boxed{r = \dfrac{-1 - \sqrt{15}i}{2}}$

KONPLEXUAK

$$\left.\begin{aligned} y_1 &= e^{-x/2}\cos\left(\tfrac{\sqrt{15}}{2}x\right) \\ y_2 &= e^{-x/2}\sin\left(\tfrac{\sqrt{15}}{2}x\right) \end{aligned}\right\} \; y_H = C_1 e^{-x/2}\cos\left(\tfrac{\sqrt{15}}{2}x\right) + C_2 e^{-x/2}\sin\left(\tfrac{\sqrt{15}}{2}x\right)$$

② $\boxed{\text{KIM}}$

$q_1 = \dfrac{e^x}{2}$

$y_1 = Ae^x$

$$\left.\begin{aligned} y_1 &= Ae^x \\ y_1' &= Ae^x \\ y_1'' &= Ae^x \end{aligned}\right| \; \begin{aligned} &Ae^x + Ae^x + 4Ae^x = e^x \\ &6Ae^x = e^x ; \; \boxed{A = 1/6} \end{aligned}$$

$q_2 = \dfrac{e^{-x}}{2}$

$y_2 = Be^{-x}$

$$\left.\begin{aligned} y_2 &= Be^{-x} \\ y_2' &= -Be^{-x} \\ y_2'' &= Be^{-x} \end{aligned}\right| \; \begin{aligned} &\cancel{Be^{-x} - Be^{-x}} + 4Be^{-x} = e^{-x} ; \\ &\boxed{B = 1/4} \end{aligned}$$

$\boxed{y_1 = \dfrac{e^x}{6}}$

$\boxed{y_2 = \dfrac{e^{-x}}{4}}$

↓ HORTAZ

$$y_{SO} = C_1 e^{-x/2}\cos\left(\tfrac{\sqrt{15}}{2}x\right) + C_2 e^{-x/2}\sin\left(\tfrac{\sqrt{15}}{2}x\right) + \frac{e^x}{6} - \frac{e^{-x}}{4}$$

c) $y'' + 3$ ...

① HO...

c) $y'' + 3y' = \underbrace{2x^4}_{g_1} + \underbrace{x^2 e^{-3x}}_{g_2} + \underbrace{\sin 3x}_{g_3}$

① HOMOGENEOA

$r^2 + 3r = 0; \quad r(r+3) = 0 \to \boxed{r=0}, \ \boxed{r=-3}$

$y_1 = e^{0x} = 1$

$y_2 = e^{-3x}$

$y_H = C_1 + C_2 e^{-3x}$

② KIM

$\boxed{g_1} = 2x^4$ 4. mailako polinomioa

$y_1 = x \cdot (Ax^4 + Bx^3 + Cx^2 + Dx + E)$

$\boxed{g_2} = x^2 e^{-3x}$

$y_2 = x \cdot (Fx^2 + Gx + H) e^{-3x}$

$\boxed{g_3}$

$y_3 = (I \cos(3x) + J \sin(3x))$

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ADIB.

$$y^{(4)} + 4y'' = \underbrace{\sin(2x)}_{g_1} + \underbrace{xe^x}_{g_2} + \underbrace{4}_{g_3}$$

KEREDA LEH + g taulan } $\boxed{KIM}$

$L[y_1] = g_1$

$L[y_2] = g_2$

$L[y_3] = g_3$

$L[y = y_1 + y_2 + y_3] = g = g_1 + g_2 + g_3$

- HOMOGENEOA PLANTEATU

$L[y] = 0$

$\hookrightarrow y^{(4)} + 4y'' = 0 \xrightarrow{\text{INDIZE EKUAZIOA}} r^4 + 4r^2 = 0;\ r^2(r^2+4) = 0$

AUTOBALIOAK $\hookrightarrow$ { $r = 0$ BIKOITZA ; $r = \pm 2i$ }

① ERRO KONPLEXUAK $\longrightarrow$ $\boxed{r = a \pm bi}$

$\boxed{r = \pm 2i}$ { $y_1 = \cos(2x)$; $y_2 = \sin(2x)$ } $\longrightarrow y = e^{rt} = e^{at}(\cos(bt) + i\sin(bt))$

② ERRO ERREALA

$\boxed{r = 0 \text{ BIKOITZA}}$ { $y_3 = e^0 = 1$; $y_4 = xe^0 = x$ }

HOMOGENEOA (y_H)

$$\boxed{y_H = C_1 + C_2 x + C_3\cos(2x) + C_4\sin(2x)}$$

- y_P BILATU

(1) $L[y_1] = g_1 = \sin(2x)$ (KIM TAULA)

$y_1 = (A\cos(2x) + B\sin(2x))x^s$ ~~s=0~~ ; $s = 1$ ✓

$\boxed{s=1}$ $\hookrightarrow y_1 = (A\cos(2x) + B\sin(2x))x$

- $y_1' =$
- $y_1'' =$
- $y_1''' =$
- $y_1^{(4)} =$

$\hookrightarrow y_1^{(4)} + 4y_1'' = \sin(2x) \xrightarrow{\text{Adierazpenea ordezkatu}} -16B\cos(2x) + 16A\sin(2x) = \sin(2x)$

(2) $L[y_2] = g_2 = xe^x$ (KIM TAULA)

$y_2 = (A+Bx)\, e^x\, x^s$ $\quad [\, s=0 \checkmark$

$\hookrightarrow y_2 = Ae^x + Bxe^x$

- $y_2' =$
- $y_2'' =$
- $y_2''' =$
- $y_2^{(IV)} =$

$\hookrightarrow y_2^{(IV)} + 4y_2'' = xe^x \xrightarrow[\text{ordezkatu}]{\text{Adierazpenak}} e^x\,[(5A+12B)+5Bx] = xe^x$

Adierazpena bete dadin $\begin{cases} B = 1/5 \\ A = -12/25 \end{cases} \longrightarrow \boxed{y_2 = \left(-\frac{12}{25} + \frac{1}{5}x\right) e^x}$

(3) $L[y_3] = g_3 = 4$ (KIM TAULA)

$y_3 = Ax^s$ $\quad \left[\begin{array}{l} s=0 \times \\ s=1 \times \\ s=2 \checkmark \end{array}\right.$

$\hookrightarrow y_3 = Ax^2$

- $y_3' =$
- $y_3'' =$
- $y_3''' =$
- $y_3^{(IV)}$

$\hookrightarrow y_3^{(IV)} + 4y_3'' = 4 \xrightarrow[\text{ordezkatu}]{\text{Adierazpenak}} 8A = 4$

Adierazpena bete dadin $\{ A = 1/2 \longrightarrow \boxed{y_3 = x^2/2}$

$$\boxed{y_{SO} = \underbrace{C_1 + C_2x + C_3\cos(2x) + C_4\sin(2x)}_{y_H} + \underbrace{\frac{x\cos(2x)}{16} + \left(-\frac{12}{25} + \frac{1}{5}x\right)e^x + \frac{x^2}{2}}_{y_P}}$$

ADIB.

$y' + 5y = e^{-5x}$

↓ INDIZE EKUAZIOA $\quad y' + 5y = 0$

$r + 5 = 0$; $\boxed{r = -5}$ ⟶ $\boxed{y_H = Ce^{-5x}}$ HOMOGENEOA

• y_p

$L[y] = g = e^{-5x}$ (KIM TAULA)

$y_1 = Ae^{-5x}x^{s} \overset{\boxed{s=1}}{=} Axe^{-5x}$

$y_1' =$

↳ $y' + 5y = e^{-5x} \xrightarrow[\text{ordezkatu}]{\text{Adierazpenak}} Ae^{-5x}(1-5x) + 5Axe^{-5x} = e^{-5x}$;

$Ae^{-5x} - 5xAe^{-5x} + 5Axe^{-5x} = e^{-5x}$; $\quad Ae^{-5x} = e^{-5x} \longrightarrow \boxed{A=1}$

$y_{SO} = \underbrace{Ce^{-5x}}_{y_H} + \underbrace{xe^{-5x}}_{y_p}$

17. 17.

A) $y'' + y' - 2y = 0$ KEKEDALH (HOMOGENEOA)

Indize ekuazioa ↳ $r^2 + r - 2 = 0$

$r = \dfrac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \dfrac{-1 \pm 3}{2}$ → $\boxed{r = -2}$, $\boxed{r = 1}$ AUTOBALIO SINPLEAK

$\boxed{r_1 = 1} \longrightarrow \boxed{y_1 = e^{x}}$

$\boxed{r_2 = -2} \longrightarrow \boxed{y_2 = e^{-2x}}$

SOLUZIO OROKORRA

$\boxed{y_{SO} = C_1 e^{x} + C_2 e^{-2x}}$

HASTAPEN BALDINTZAK APLIKATU $\quad \begin{cases} y(0) = 1 \\ y'(0) = 1 \end{cases}$

$y_{SO} = 1 \cdot e^{x} + 0 \cdot e^{-2x}$; $\boxed{y_{SO} = e^{x}}$

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b) $y'' - 6y' + 9y = 0$

↓ Indize ekuazioa

$r^2 - 6r + 9 = 0 \longrightarrow \boxed{r = 3}$ BIKOITZA

$$\begin{cases} y_1 = e^{3x} \\ y_2 = xe^{3x} \end{cases}$$

SOLUZIO OROKORRA

$y_{SO} = C_1 e^{3x} + C_2 x e^{3x}$

d) $y'' + 8y' - 9y = 0$

↓ Indize ekuazioa

$r^2 + 8r - 9 = 0 \longrightarrow \begin{cases} \boxed{r = 2} \\ \boxed{r = -1 \pm \sqrt{3}} \end{cases}$

$\boxed{r = 2}$

$y_1 = e^{2x}$

$\boxed{r = -1 \pm \sqrt{3}}$ ERRO KONPLEXUA $\longrightarrow \boxed{r = a \pm ib}$

$e^{ax}(\cos(bx) + i\sin(bx)) = e^{-x}(\cos(\sqrt{3}x) + i\sin(\sqrt{3}x))$

$y_2 = e^{-x}\cos(\sqrt{3}x)$

$y_3 = e^{-x}\sin(\sqrt{3}x)$

SOLUZIO OROKORRA

$y_{SO} = e^{2x} + e^{-x}\cos(\sqrt{3}x) + e^{-x}\sin(\sqrt{3}x)$

c) $y''' - y'' - y' + y = 0$

Indize ekuazioa $\rightarrow r^3 - r^2 - r + 1 = 0 \longrightarrow \begin{cases} \boxed{r = -1} \\ \boxed{r = 1} \text{ BIKOITZA} \end{cases}$

$\boxed{r = -1}$

$y_1 = e^{-x}$

$\boxed{r = 1}$ BIKOITZA

$y_2 = e^{x}$

$y_3 = xe^{x}$

SOLUZIO OROKORRA

$y_{SO} = C_1 e^{-x} + C_2 e^{x} + C_3 x e^{x}$

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