ADIB.

- $f(t)=1$

$$\hookrightarrow \mathcal{L}[1](s) = \int_0^\infty 1\, e^{-st}\,dt \overset{s>0}{=} \left[\frac{e^{-st}}{-s}\right]_0^\infty = 0-\left(\frac{1}{-s}\right) = \frac{1}{s} \quad \boxed{\forall s>0}$$

- $$\mathcal{L}[t^n](s) = \int_0^\infty t^n e^{-st}\,dt = \left\{ \begin{array}{ll} u=t^n & du=nt^{n-1} \\ dv=e^{-st}dt & v=\frac{e^{-st}}{-s} \end{array} \right\} =$$

$$= \left[t^n\cdot\left(\frac{e^{-st}}{-s}\right)\right]_0^\infty - \int_0^\infty \left(\frac{e^{-st}}{-s} n\cdot t^{n-1}\right)dt =$$

$$= \left[t^n\cdot\left(\frac{e^{-st}}{-s}\right)\right]_0^\infty + \frac{n}{s}\underbrace{\boxed{\int_0^\infty e^{-st}t^{n-1}\,dt}}_{\mathcal{L}[t^{n-1}](s)} = (0-0)+\frac{n}{s}\mathcal{L}[t^{n-1}](s) =$$

$$= \frac{n(n-1)}{s^2}\mathcal{L}[t^{n-2}](s) = \dots = \frac{n!}{s^n}\mathcal{L}[1](s) = \frac{n!}{s^{n+1}}, \quad \boxed{\forall s>0}$$

- $\mathcal{L}[t](s) = \frac{1}{s^2}, \quad \forall s>0$
- $\mathcal{L}[t^2](s) = \frac{2}{s^3}, \quad \forall s>0$

AURRETIK FROGATUTAKO ADIERAZPENA

$$\left.\right\} \mathcal{L}[t^n](s) = \frac{n!}{s^{n+1}}, \quad \forall s>0$$

- $\times\ \mathcal{L}[f]=F$

  $\times\ \mathcal{L}[g]=G$

$$\left.\right\} \rightarrow \mathcal{L}[\alpha f+\beta g] = \alpha F+\beta G \qquad \forall \alpha,\beta\in\mathbb{R},\mathbb{C}$$

- $\mathcal{L}[8+2t-t^2] = \frac{8}{s}+\frac{2}{s^2}-\frac{2}{s^3} \qquad \forall s>0$

- $$\mathcal{L}[e^{at}](s) = \int_0^\infty e^{at}\cdot e^{-st}\,dt = \int_0^\infty e^{t(a-s)}\,dt \overset{a-s<0}{=} \left[\frac{e^{t(a-s)}}{a-s}\right]_0^\infty =$$

$$= 0-\frac{1}{a-s} = -\frac{1}{a-s} \qquad \forall s>a$$

ADIB.

1. $\mathcal{L}[t^5 + 7t^3 + 2t + 1] = \frac{5!}{s^6} + 7\frac{3!}{s^4} + 2\frac{1}{s^2} + \frac{1}{s}$

2. $\mathcal{L}[e^{5t}] = \frac{1}{s-5}$

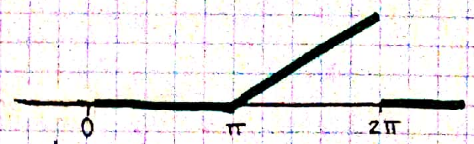
3. $\mathcal{L}[e^{-2t}] = \frac{1}{s+2}$

4. $\mathcal{L}[\sin(3t)] = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$

5. $\mathcal{L}[\cos(4t)] = \frac{s}{s^2+4^2} = \frac{s}{s^2+16}$

ADIB.

$$f(t) = \begin{cases} 0 & t<\pi \\ t-\pi & \pi<t<2\pi \\ 0 & t>2\pi \end{cases}$$



$\mathcal{L}\downarrow$ $f(t) = (t-\pi)\cdot(H_\pi(t) - H_{2\pi}(t)) = (t-\pi)\cdot(H(t-\pi) - H(t-2\pi))$

$$\mathcal{L}[f] = \underbrace{\mathcal{L}[(t-\pi)\cdot H(t-\pi)]}_{A} - \underbrace{\mathcal{L}[(t-\pi)\cdot H(t-2\pi)]}_{B}$$

A $\dfrac{e^{-s\pi}}{s^2}$

(t-2π) lortu behar dugu

B Formularen adierazpena ez duenez, aldatu beharra dago

$(t-\pi)H_{2\pi} = (t-\pi-\pi+\pi)H_{2\pi} = (t-2\pi+\pi)H_{2\pi} = (t-2\pi)H_{2\pi} + \pi H_{2\pi}$;

$$\mathcal{L}[(t-2\pi)H_{2\pi}] + \pi\mathcal{L}[H_{2\pi}] = \frac{e^{-2s\pi}}{s^2} + \pi\frac{e^{-2s\pi}}{s}$$

↓ HORTAZ

$$\mathcal{L}[f] = \frac{e^{-s\pi}}{s^2} + \frac{e^{-2s\pi}}{s^2} + \pi\frac{e^{-2s\pi}}{s}$$

OHARRA!

$t \xrightarrow{\mathcal{L}} 1/s^2$

$(t-a)H_\pi \longrightarrow \dfrac{e^{-s\pi}}{s^2}$

ADIB.

$$\begin{cases} y'' + 3y' + 2y = \delta_a = \delta(t-a) \\ y(0) = y'(0) = 0 \end{cases}$$ (HASTAPEN BALDINTZAKO PROBLEMA)

Laplace-en transformatua aplikatu

$\mathcal{L}[y'' + 3y' + 2y] = \mathcal{L}[\delta_a]$;

$s^2Y(s) - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{0} + 3(sY(s) - \cancel{y(0)}^{0}) + 2Y(s) = e^{-as}$;

$s^2Y(s) + 3sY(s) + 2Y(s) = e^{-as}$; $Y(s)(s^2+3s+2) = e^{-as}$;

$\mathcal{L}^{-1}\downarrow$ aplikatu

$$Y(s) = \frac{e^{-as}}{s^2+3s+2}$$

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[\frac{e^{-as}}{s^2+3s+2}\right] = \mathcal{L}^{-1}\left[e^{-as}\cdot\frac{1}{s^2+3s+2}\right]$$

(a) FRAKZIO SINPLEETAN BANATU

$$F(s) = \frac{1}{s^2+3s+2} = \frac{1}{(s+2)\cdot(s+1)} \overset{(a)}{=} \frac{1}{s+1} - \frac{1}{s+2}$$

$\mathcal{L}\downarrow$

$$f(t) = \mathcal{L}[F(s)] = \mathcal{L}\left[\frac{1}{s+1}\right] - \mathcal{L}\left[\frac{1}{s+2}\right] = e^{-t} - e^{-2t}$$

- $y(t) = \mathcal{L}^{-1}[Y(s)] = H(t-a)\,f(t-a) = H(t-a)\left[e^{-t+a} - e^{-2t+2a}\right] =$

$$= \begin{cases} 0 & t<a \quad H(t-a)=0 \\ e^{-t+a} - e^{-2(t-a)} & t>a \end{cases}$$

ADIB.

$$\begin{cases} y'' + y = 2t \\ y(\pi/4) = \pi/2 \\ y'(\pi/4) = 2 - \sqrt{2} \end{cases} \Big\} \text{ HASTAPEN BALDINTZAK}$$

LAPLACE TRANSFORMATUA ERABILIZ:

$\mathcal{L}[y''+y] = \mathcal{L}[2t]$; $s^2 Y(s) - s y(0) - y'(0) + Y(s) = 2 \cdot \frac{1}{s^2}$;

$Y(s)(s^2+1) = \frac{2}{s^2} + s y(0) + y'(0)$; $Y(s) = \dfrac{2/s^2 + s y(0) + y'(0)}{(s^2+1)}$;

$Y(s) = \dfrac{2}{s^2(s^2+1)} + \dfrac{s y(0)}{(s^2+1)} + \dfrac{y'(0)}{(s^2+1)}$ NON $\begin{cases} y(0) = \alpha \\ y'(0) = \beta \end{cases}$ EZEZAGUNAK

ORAIN $\mathcal{L}^{-1}$ APLIKATU $y(t)$ KALKULATZEKO

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[\frac{2}{s^2(s^2+1)}\right]^{*} + \alpha \underbrace{\mathcal{L}^{-1}\left[\frac{s}{(s^2+1)}\right]}_{\cos t} + \beta \underbrace{\mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right]}_{\sin t}$$

* DESKONPOSAKETA

$$\frac{2}{s^2(s^2+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+1};$$

$$2 = As(s^2+1) + B(s^2+1) + (Cs+D)s^2;$$

$$2 = s^3(A+C) + s^2(B+D) + s(A) + B$$

$$\begin{cases} A+C=0 & \boxed{C=0} \\ B+D=0; & \boxed{D=-2} \\ \boxed{A=0} \\ \boxed{B=2} \end{cases}$$

HORTAZ

$$\frac{2}{s^2(s^2+1)} = \frac{2}{s^2} - \frac{2}{s^2+1}$$

$$\mathcal{L}^{-1}\left[\frac{2}{s^2} - \frac{2}{s^2+1}\right] = 2\underbrace{\mathcal{L}^{-1}\left[\frac{1}{s^2}\right]}_{t} - 2\underbrace{\mathcal{L}^{-1}\left[\frac{1}{s^2+1}\right]}_{\sin t}$$

$$y(t) = 2t - 2\sin t + \alpha\cos t + \beta\sin t$$

* KONSTANTE BATEKIN ELKARTU

$$\boxed{y(t) = 2t + \alpha\cos t + \gamma\sin t}$$

HASTAPEN BALDINTZAK ORDEZKATU

$y(\pi/4) = 2\cdot\frac{\pi}{4} + \alpha\cos\pi/4 + \gamma\sin\pi/4 = \pi/2$; $\frac{\pi}{2} + \frac{\sqrt{2}}{2}(\alpha+\gamma) = \pi/2$

$y'(\pi/4) = \frac{\pi}{4} - \alpha\sin\pi/4 + \gamma\cos\pi/4 = 2-\sqrt{2}$; $2 + \frac{\sqrt{2}}{2}(-\alpha+\gamma) = 2-\sqrt{2}$

$\begin{cases} \gamma = -1 \\ \alpha = 1 \end{cases} \longrightarrow \boxed{y(t) = 2t - \sin t + \cos t}$

ARIKETAK

1.

A) $\begin{cases} y'' + 2y' + 5y = 0 \\ y(0) = 2 \\ y'(0) = -1 \end{cases}$

LAPLACE-en TRANSFORMATUA

$$\mathcal{L}[y'' + 2y' + 5y] = 0 \;;\; \mathcal{L}[y''] + 2\mathcal{L}[y'] + 5\mathcal{L}[y] = 0;$$

$$s^2 Y(s) - sy(0) - y'(0) + 2\cdot(sY(s) - y(0)) + 5Y(s) = 0;$$

$\downarrow$ $Y(s)$ BAKANDU

$$s^2Y(s) - sy(0) - y'(0) + 2sY(s) - 2y(0) + 5Y(s) = 0;$$

$$Y(s)(s^2 + 2s + 5) = sy(0) + y'(0) + 2y(0);$$

$$\boxed{Y(s) = \frac{2s+3}{s^2+2s+5}}$$

$s^2 + 2s + 5$ FAKTORIZATU

$$s = \frac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot 5}}{2\cdot 1} = \cancel{-2 \pm \sqrt{\ominus}}$$

IDENTITATE NABARMENAK KONTUAN IZANIK

$$s^2 + 2s + 5 - 4 + 4 = (s+1)^2 + 4 = (s+1)^2 + 2^2$$

$\downarrow$ HORTAZ

$$Y(s) = \frac{2s+3}{(s+1)^2+2^2} = \frac{2s}{(s+1)^2+2^2} + \frac{3}{(s+1)^2+2^2} =$$

$$= 2\,\frac{s+1}{(s+1)^2+2^2} - 1 + \frac{1}{2}\,\frac{2}{(s+1)^2+2^2}$$

$\downarrow$ $y(t)$ lortzeko

$$\mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[2\,\frac{s+1}{(s+1)^2+2^2} + \frac{1}{2}\,\frac{2}{(s+1)^2+2^2}\right] =$$

$$= 2\mathcal{L}^{-1}\left[\frac{s+1}{(s+1)^2+2^2}\right] - \frac{1}{2}\mathcal{L}^{-1}\left[\frac{2}{(s+1)^2+2^2}\right];$$

$$y(t) = 2e^{-t}\cos(2t) + \frac{1}{2}\sin(2t)e^{-t}$$

B) $\begin{cases} y^{(iv)} - 4y = 0 \\ y(0) = 1 \\ y'(0) = 0 \\ y''(0) = -2 \\ y'''(0) = 0 \end{cases}$

LAPLACE-en TRANSFORMATUA aplikatu

$$\mathcal{L}[y^{(iv)} - 4y] = 0 \; ; \; \mathcal{L}[y^{(iv)}] - 4\mathcal{L}[y] = 0;$$

$$s^4 Y(s) - s^3 y(0) - s^2 y'(0) - s y''(0) - y'''(0) - 4Y(s) = 0$$

↓ $Y(s)$ BAKANDU

$$Y(s)(s^4 - 4) = s^3 y(0) + \cancel{s^2 y'(0)} + s y''(0) - \cancel{y'''(0)};$$

$$\boxed{Y(s) = \frac{s^3 - 2s}{s^4 - 4}}$$

↓ FAKTORIZATU

$$Y(s) = \frac{s\cancel{(s^2 - 2)}}{\cancel{(s^2 - 2)}(s^2 + 2)} = \frac{s}{s^2 + 2}$$

↓ $\mathcal{L}^{-1}$ aplikatu

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}\left[\frac{s}{s^2 + 2}\right] = \cos(\sqrt{2}\,t)$$

C) $\begin{cases} y'' - 2y' + 2y = \cos t \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$

LAPLACE-en TRANSFORMATUA aplikatu

$$\mathcal{L}[y'' - 2y' + 2y] = \mathcal{L}[\cos t]; \; \mathcal{L}[y''] - 2\mathcal{L}[y'] + 2\mathcal{L}[y] = \mathcal{L}[\cos t];$$

$$s^2 Y(s) - s y(0) - y'(0) - 2[sY(s) - y(0)] + 2Y(s) = \frac{s}{s^2 + 1};$$

↓ $Y(s)$ BAKANDU

$$Y(s)(s^2 - 2s + 2) = \frac{s}{s^2 + 1} + s - 2; \; Y(s) = \frac{s}{(s^2 + 1)(s^2 - 2s + 2)} + \frac{s - 2}{(s^2 - 2s + 2)}$$

$$s^2 - 2s + 2 = (s - 1)^2 + 1$$

$$\frac{s}{(s^2 + 1)(s^2 - 2s + 2)} = \frac{As + B}{(s^2 + 1)} + \frac{Cs + D}{(s^2 - 2s + 2)};$$

$$s = (As + B)(s^2 - 2s + 2) + (Cs + D)(s^2 + 1);$$

$$s = s^3(A + C) + s^2(-2A + B + D) + s(2A - 2B + C) + 2B + D$$

3.

A) $\begin{cases} y'' + 4y = \sin t - H_{2\pi}(t)\sin(t-2\pi) \\ y(0)=0 \\ y'(0)=0 \end{cases}$

LAPLACE -en TRANSFORMATUA aplikatu

$$\mathcal{L}[y''+4y] = \mathcal{L}[\sin t - H_{2\pi}(t)\sin(t-2\pi)];$$

$$\mathcal{L}[y''] + 4\mathcal{L}[y] = \mathcal{L}[\sin t] - \mathcal{L}[H_{2\pi}(t)\sin(t-2\pi)];$$

$$s^2Y(s) - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{0} + 4Y(s) = \frac{1}{s^2+1} - e^{-2\pi s}\frac{1}{s^2+1}$$

$$Y(s)(s^2+4) = \frac{1}{s^2+1} - e^{-2\pi s}\frac{1}{s^2+1};$$

$$Y(s) = \frac{1}{(s^2+1)(s^2+4)} - e^{-2\pi s}\frac{1}{(s^2+1)(s^2+4)};$$

$$Y(s) = \frac{(1-e^{-2\pi s})}{(s^2+1)(s^2+4)}$$

FAKTORIZAZIOA

$$\frac{1-e^{-2\pi s}}{(s^2+1)(s^2+4)} = \frac{As+B}{(s^2+1)} + \frac{Cs+D}{(s^2+4)};$$

$$1-e^{-2\pi s} = (As+B)(s^2+4) + (Cs+D)(s^2+1);$$

$$1-e^{-2\pi s} = s^3(A+C) + s^2(B+D) + s(4A+C) + 4B + D$$

$A+C=0$; $A=-C$ $\boxed{A=0}$ $\boxed{D=\frac{e^{-2\pi s}-1}{3}}$

$B+D=0$; $B=-D$

$4A+C=0$; $C=-4A$; $C=4C$; $\boxed{C=0}$

$4B+D=1-e^{-2\pi s}$; $4B-B=1-e^{-2\pi s}$; $\boxed{B=\frac{1-e^{-2\pi s}}{3}}$

$$y(t) = \mathcal{L}^{-1}\left[\frac{1-e^{-2\pi s}}{3(s^2+1)}\right] + \mathcal{L}^{-1}\left[\frac{e^{-2\pi s}-1}{3(s^2+4)}\right] =$$

$$= \frac{1-H_{2\pi}}{3}\sin t + \frac{1}{2}\frac{H_{2\pi}-1}{3}\sin(2t)$$

4.

A) $\begin{cases} y'' + 2y' + y = \delta(t) + H_{2\pi}(t) \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$

LAPLACE-en TRANSFORMATUA

$$\mathcal{L}[y'' + 2y' + y] = \mathcal{L}[\delta(t) + H_{2\pi}(t)];$$

$$s^2 Y(s) - s\cancel{y(0)} - y'(0) + 2sY(s) - \cancel{2y(0)} + Y(s) = 1 + \frac{e^{-2\pi s}}{s}$$

$$Y(s)(s^2 + 2s + 1) = 1 + \frac{e^{-2\pi s}}{s} + 1 \; ; \; Y(s) = \underbrace{\frac{2}{s^2+2s+1}}_{A} + \underbrace{\frac{e^{-2\pi s}}{(s^2+2s+1)s}}_{B}$$

A $\mathcal{L}^{-1}\left[\frac{2}{(s+1)^2}\right] = 2te^{-t}$

B $\frac{e^{-2\pi s}}{s(s+1)^2} = \frac{A}{s} + \frac{Bs+C}{(s+1)^2};$

$$e^{-2\pi s} = A(s+1)^2 + s(Bs+C);$$

$$e^{-2\pi s} = s^2(A+B) + s(2A+C) + (A);$$

$$\begin{cases} \boxed{A = e^{-2\pi s}} \\ A + B = 0; \; \boxed{B = -e^{-2\pi s}} \\ 2A + C = 0; \; C = -2A; \; \boxed{C = -2e^{-2\pi s}} \end{cases}$$

$$y(t) = 2te^{-t} + \mathcal{L}^{-1}\left[\frac{e^{-2\pi s}}{s} - \frac{se^{-2\pi s} - 2e^{-2\pi s}}{(s+1)^2}\right] =$$

$$= 2te^{-t} + H_{2\pi}(1 - e^{-t} - e^{-t}t)$$

$$y(t) = 2te^{-t} + H_{2\pi}\left[1 - e^{-t+2\pi}(1 + t + 2\pi)\right]$$

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7.

A) $F(s) = \dfrac{3}{s^2+4}$

$$\mathcal{L}^{-1}[F(s)] = \left[3 \frac{1}{s^2+4} \frac{2}{2}\right] = \frac{3}{2}\sin(2t)$$

B) $F(s) = \dfrac{4}{(s-1)^3}$

$$\mathcal{L}^{-1}[F(s)] = \left[2 \frac{2}{(s-1)^3}\right] = 2e^t t^2$$

C) $F(s) = \dfrac{2}{s^2+3s-4}$

$$s^2+3s-4=0 \longrightarrow s = \frac{-3 \pm \sqrt{3^2-4\cdot 1\cdot(-4)}}{2\cdot 1} = \frac{-3\pm 5}{2} \quad \boxed{s=-4} \quad \boxed{s=1}$$

$$\frac{2}{(s+4)(s-1)} = \frac{A}{(s+4)} + \frac{B}{(s-1)}$$

$$2 = A(s-1) + B(s+4) \,;\quad 2 = s(A+B) - A + 4B$$

$$\begin{cases} A+B=0 \,;\ \underline{A=-B} \,;\ \boxed{A=-2/5} \\ -A+4B=2 \,;\ 5B=2 \,;\ \boxed{B=2/5} \end{cases}$$

$$-\frac{2}{5}\frac{1}{s+4} + \frac{2}{5}\frac{1}{s-1}$$

$\mathcal{L}^{-1} \downarrow$

$$-\frac{2}{5}e^{-4t} + \frac{2}{5}e^{t} \,;\quad \mathcal{L}^{-1}[F(s)] = \frac{2}{5}\left(-e^{-4t}+e^{t}\right)$$

D) $F(s) = \dfrac{3s}{s^2-s-6}$

$$s^2-s-6=0 \longrightarrow s = \frac{-(-1)\pm\sqrt{(-1)^2-4\cdot 1\cdot(-6)}}{2\cdot 1} = \frac{1\pm 5}{2} \quad \boxed{s=3} \quad \boxed{s=-2}$$

$$\frac{3s}{(s-3)(s+2)} = \frac{A}{s-3} + \frac{B}{s+2}$$

$$3s = A(s+2) + B(s-3) \,;\quad 3s = s(A+B) + 2A - 3B$$

$$\begin{cases} A+B=3 \,;\ \underline{A=3-B} \,;\ A = 3-\frac{6}{5} \,;\ \boxed{A=9/5} \\ 2A-3B=0 \,;\ 6-2B-3B=0 \,;\ 6-5B=0 \,;\ \boxed{B=6/5} \end{cases}$$

$$\frac{9}{5}\frac{1}{s-3} + \frac{6}{5}\frac{1}{s+2}$$

$\mathcal{L}^{-1} \downarrow$

$$\frac{9}{5}e^{3t} + \frac{6}{5}e^{-2t}$$