③

A) $\begin{cases} y' = \dfrac{2y^3}{x} \\ y(0) = 1 \end{cases} \longrightarrow x y' = 2y^3$ EDO $x\,dy = 2y^3\,dx$ BANANGARRIA!

- $f(x,y) = \dfrac{2y^3}{x}$
- $\dfrac{\partial f}{\partial y} = \dfrac{6y^2}{x}$

EZ DIRA JARRAITUAK (0,1) PUNTUAN

↳ EZ DA PICCARD BETEKO

$\int \dfrac{dy}{y^3} = \int \dfrac{2\,dx}{x}$; $\dfrac{y^{-2}}{-2} = 2\,L|x| + K$; $\dfrac{1}{y^2} = -4\,L|x| + K$;

$\dfrac{1}{y^2} = L\left|\dfrac{1}{x^4}\right| + K$, $K \in \mathbb{R}$

y(0)=1 ordezkatuz ↳ $\dfrac{1}{1} = L\left|\dfrac{1}{0^4}\right| + K$ EZ DA HASTAPEN BALDINTZA BETETZEN

↳ PROBLEMAK EZ DU SOLUZIORIK

↓ 3A) ARIKETA BAINA PUNTU EZBERDIN BATEAN

$\begin{cases} y' = \dfrac{2y^3}{x} \\ y(1) = 2 \end{cases}$

- $f(x,y) = \dfrac{2y^3}{x}$
- $\dfrac{\partial f}{\partial y} = \dfrac{6y^2}{x}$

Jarraituak dira (1,2) puntuan eta (1,2) puntuaren ingurune batean

↳ $\exists$ soluzio bakar bat (PICCARD)

$$\int \frac{dy}{y^3} = \int \frac{2dx}{x} \; ; \; \frac{1}{y^2} = L\left|\frac{1}{x^4}\right| + C$$

Hastapen baldintza aplikatuz:

$$\frac{1}{2^2} = \cancel{L\left|\frac{1}{1^4}\right|}^{\to 0} + C \; ; \; \boxed{C = 1/4}$$

$$y^2 = \frac{1}{-4\,L|x| + 1/4} = \frac{4}{1-16L|x|} \; ; \; y = \sqrt{\frac{4}{1-16|x|}} = \frac{2}{\sqrt{1-16|x|}} \qquad \boxed{0<x<e^{1/16}}$$

JARRAITUA NON?

$$1-16|x| > 0 \; ; \; 16\,L|x| < 1 \; ; \; L|x| < 1/16 \; ; \; x < e^{1/16}$$

PAM → Parametroen aldakuntzako metodoa

ADIB.

$$x y' + 2y = \underbrace{x^2 - x + 1}_{q(x)}$$

(1) $xy' + 2y = 0$ (Elkarturiko EDA lineal homogeneoa $L[y]=0$)

$$y' = \frac{dy}{dx} = -\frac{2y}{x}; \quad \frac{dy}{y} = -\frac{2dx}{x}; \quad \int \frac{dy}{y} = -\int \frac{2dx}{x}; \quad L|y| = -2L|x| + K;$$

$$L|y| = L|1/x^2| + K; \quad y = e^{L|1/x^2| + K}; \quad y = 1/x^2 \cdot e^K;$$

$$\boxed{y_h = \frac{K}{x^2}} \qquad K \in \mathbb{R}$$

(2) $y_p = \dfrac{K(x)}{x^2} = x^{-2} \cdot K(x)$ NON $y'_p = \dfrac{K'(x)x^2 - K(x) \cdot 2x}{x^4} = \dfrac{K'(x)}{x^2} - \dfrac{2K(x)}{x^3}$

↓ EDA OSOAN ordezkatu $xy' + 2y = x^2 - x + 1$

$$x \cdot \left(\frac{K'(x)}{x^2} - \frac{2K(x)}{x^3} \right) + 2\frac{K(x)}{x^2} = x^2 - x + 1;$$

$$\frac{K'(x)}{x} - \cancel{\frac{2K(x)}{x^2}} + \cancel{\frac{2K(x)}{x^2}} = x^2 - x + 1; \quad K'(x) = x^3 - x^2 + x$$

$K'(x)$ INTEGRATU ⟶ $K(x) = \dfrac{x^4}{4} - \dfrac{x^3}{3} + \dfrac{x^2}{2}$

SOLUZIOA

$$\boxed{y_{s.o} = \frac{x^2}{4} - \frac{x}{3} + \frac{1}{2} + \frac{K}{x^2}}$$

Gero, K-ren balioa lortzeko, hasierako baldintzako problema aplikatuko genuke, baina kasu honetan ez dugu hasierako baldintzarik eraintzatuan.

ADIB.

$(\cos t \cdot \sin t - ty^2) + y(1-t^2)y' = 0$

↳ $\underbrace{(\cos t \cdot \sin t - ty^2)}_{M} dt + \underbrace{y(1-t^2)}_{N} dy = 0 \longrightarrow M dt + N dy = 0$

$\downarrow$

$\underbrace{\frac{\partial \varphi}{dt}}_{M} + \underbrace{\frac{\partial \varphi}{\partial y}}_{N} y' = 0$

- $\frac{\partial M}{\partial y} = -2ty$
  (11)
- $\frac{\partial N}{\partial t} = -2ty$

EDA ZEHATZA

Soluzioa kurben familia bat da.

$\varphi(t,y) = C$, NON $C \in \mathbb{R}$

- $\varphi(t,y)$ LORTU $\longrightarrow$ N INTEGRATU y-REKIKO $\left(N(t,y) = \frac{\partial \varphi}{\partial y} \text{ BAITA} \right)$

↳ $\int N dy = \int d\varphi = \varphi$

$\varphi(t,y) = \int y(1-t^2)\, dy + h(t) = (1-t^2)\frac{y^2}{2} + \underbrace{h(t)}$

LORTU BEHAR DUGUNA

↓ DERIBATU t-rekiko

$M = \frac{\partial \varphi}{\partial t} = -\frac{\cancel{2}ty^2}{\cancel{2}} + h'(t); \; -ty^2 + h'(t) - M = 0;$

$h'(t) = \cancel{ty^2} + (\cos t \cdot \sin t - \cancel{ty^2}); \; \boxed{h'(t) = \cos t \cdot \sin t}$

↓ h'(t) INTEGRATU h(t) LORTZEKO

$h(t) = \int h'(t)\, dt = \frac{\sin^2 t}{2}$

SOLUZIOA

$\varphi(t,y) = (1-t^2)\frac{y^2}{2} + \frac{\sin^2 t}{2} = C \quad C \in \mathbb{R}$

$\boxed{\varphi(t,y) = y^2(1-t^2) + \sin^2 t = C \quad C \in \mathbb{R}}$

$\downarrow$

KURBEN FAMILIA

ADIB.

$$y' = \frac{x^2+3y^2}{2xy} = \frac{1+3(y/x)^2}{2(y/x)} = F(y/x) = F(u)$$

$$\frac{y}{x} = u \; ; \; y = ux \xrightarrow{\text{DERIBATU}} y' = u + x u'$$

y'-ren 2 adierazpenak berdindu

$$\hookrightarrow u + xu' = \frac{1+3u^2}{2u} \; ; \; u' = \frac{1+3u^2-2u^2}{2ux} \; ; \; \boxed{u' = \frac{1+u^2}{2ux}}$$

BANANGARRIA $\longrightarrow \frac{2u}{1+u^2}\, du = \frac{dx}{x}$

↓ INTEGRATU

$$\int \frac{2u}{1+u^2}\, du = \int \frac{dx}{x} \; ; \; L|1+u^2| = L|x| + C \; ; \; e^{L|1+u^2|} = e^{L|x|+C} \; ;$$

$$\boxed{1+u^2 = x \cdot K \quad , \quad K \in \mathbb{R}}$$

↓ ALDAKETA DESEGIN $\quad (u = y/x)$

$$1 + (y/x)^2 = x \cdot K \; ; \; \left(\frac{y}{x}\right)^2 = xK - 1 \; ; \; \boxed{y^2 = x^2(Kx-1)}$$

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AURREKO ADIBIDEA HASTAPEN BALDINTZAKO PROBLEMAREKIN

$$\begin{cases} y' = \dfrac{x^2 + 3y^2}{2xy} \\ y(0) = 1 \end{cases}$$

- SOLUZIORIK AL DU?

$$\frac{\partial F}{\partial y} = \frac{6y(2xy) - (x^2+3y^2)2x}{(2xy)^2} = \frac{0}{0}$$ F EZ DA JARRAITUA (0,1) PUNTUAN

HORTAZ $\longrightarrow$ PICCARD-en TEOREMAK ez digu soluziorik bermatzen

- HOMOGENEOA IZANIK EBATZI

$$y' = \frac{1 + 3(y/x)^2}{2(y/x)} \cdots \longrightarrow y^2 = x^2(kx - 1) \xrightarrow{(0,1) \text{ ORDEZTU}} 1 \neq 0$$

$\boxed{\text{EZ DAGO SOLUZIORIK}}$

⊗ INDETERMINAZIOA EMANEZ GERO $\longrightarrow$ POLARRETARA

ADIB.

$$y' = \frac{y + x}{x} \longrightarrow y' = \frac{y/x + 1}{1} = \left(\frac{y}{x}\right) + 1 = u + 1$$

$$u = y/x \longrightarrow x \cdot u = y \xrightarrow{\text{DERIBATU}} y' = u'x + u$$

y'-ren 2 adierazpenak berdindu

$$x u' + u = u + 1 \; ; \; u' = \frac{1}{x} = \frac{du}{dx} \; ; \; \frac{dx}{x} = du$$

↓ INTEGRATU

$$\int \frac{dx}{x} = \int du \; ; \; u = L|x| + C = y/x \; ; \; \boxed{y(x) = x\,L|x| + xC \quad C \in \mathbb{R}}$$

(y BAKANDU)

ADIB.

$y' = \frac{1}{x} y + \frac{1}{y} = y/x + y^{-1}$

BERNOULLI NON $\boxed{p=-1}$ $\quad U = y^{1-p} \overset{p=-1}{=} y^{1-(-1)} = y^2$

$U' = (y^2)' = 2yy' \; ; \; \boxed{y' = \frac{U'}{2y}}$

1. ORDENAKO EDA LINEALA

y'-ren 2 adierazpenak BERDINDU

$\frac{U'}{2y} = \frac{y}{x} + \frac{1}{y} \; ; \; \frac{U'}{2} = \frac{\overset{=U}{y^2}}{x} + 1 \; ; \; \frac{U'}{2} = \frac{1}{x} U + 1 \; ; \; \boxed{U' = \frac{2}{x} U + 2}$

$U' = \frac{dU}{dx} = \frac{2}{x} U \; ; \; \frac{dU}{U} = 2 \frac{dx}{x} \; ; \; \int \frac{dU}{U} = 2 \int \frac{dx}{x} \; ; \; L|U| = 2 L|x| + K \; ;$

$\boxed{U_H = x^2 K}$

PAM PARAMETROEN ALDAKUNTZARAKO METODOA

$U_p = x^2 \cdot K(x)$ NON $U'_p = 2x K(x) + x^2 K'(x)$

U'-ren 2 adierazpenak berdindu

$2x K(x) + x^2 K'(x) = \frac{2}{x} \overbrace{Kx^2}^{U} + 2 \; ; \; \cancel{2xK} + x^2 K' = \cancel{2xK} + 2 \; ; \; \boxed{K' = \frac{2}{x^2}}$

K'-ren adierazpena INTEGRATU K LORTZEKO

$K = \int K' = \int \frac{2}{x^2} = 2 \cdot (-1) \cdot x^{-1} \; ; \; \boxed{K = -\frac{2}{x}}$

$\rightarrow U_p = x^2 \cdot \left(-\frac{2}{x}\right) \; ; \; \boxed{U_p = -2x}$

$\hookrightarrow U_{SO} = Kx^2 - 2x \xrightarrow[U=y^2]{} \boxed{y^2 = Kx^2 - 2x} \quad K \in \mathbb{R}$

ADIB.

$$\begin{cases} xy' = y + (xy)^{1/2} \\ y(1) = 1 \end{cases}$$

$\left.\begin{matrix} x>0 \\ y>0 \end{matrix}\right\}$ Hau bete dadin x eta y positiboak izan behar dira, HORREGATIK $x, y > 0$ baldintza inposatu dugu.

y' BAKANDU

$$y' = \frac{y}{x} + \frac{(xy)^{1/2}}{x} = \frac{y}{x} + \left(\frac{xy}{x^2}\right)^{1/2} ; \quad \boxed{y' = \frac{y}{x} + (y/x)^{1/2}}$$

$$\rightarrow y' = y\left(\frac{1}{x}\right) + \frac{\sqrt{x}}{x} y^{1/2}$$

$p = 1/2$

BERNOULLI NON $\boxed{p = 1/2}$ $\quad U = y^{1-p} = y^{1-1/2} = y^{1/2}$

9. $f(x)$? NON $y^2 \sin(x)\,dx + y f(x)\,dy = 0$ EKUAZIO ZEHATZA eta $f(0) = 0$

$$\begin{cases} y^2 \sin(x)\,dx + y f(x)\,dy = 0 \\ f(0) = 0 \end{cases} \longrightarrow M dx + N dy = 0$$

ZEHATZA denez $\longrightarrow \boxed{\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}}$

- $\dfrac{\partial M}{\partial y} = 2y \sin x$

11

- $\dfrac{\partial N}{\partial x} = y f'(x)$

$2y \sin x = y f'(x) ; \quad f'(x) = 2 \sin x$

INTEGRATU $f(x)$ LORTZEKO

$$\boxed{f(x) = -2\cos x + K}$$

K-ren balioa lortzeko, HASTAPEN BALDINTZAKO PROBLEMA

$f(0) = -2\cos(0) + K = 0 ; \quad \boxed{K = 2}$

HORTAZ

$$\boxed{f(x) = -2\cos x + 2}$$

ENUNTZIATUKO EDA ZEHATZAN ORDEZKATUZ

$$y^2 \sin(x)\,dx + y(-2\cos x + 2)\,dy = 0$$

③

B) $\begin{cases} y' = \dfrac{y^5}{4} \\ y(0)=1 \end{cases} \longrightarrow 4y' = y^5$ EDO $\dfrac{1}{y^5}dy = \dfrac{1}{4}dx$

- $f(x,y) = \dfrac{y^5}{4}$
- $\dfrac{\partial f}{\partial y} = \dfrac{5y^4}{4}$

JARRAITUAK DIRA (0,1) PUNTUAN
↳ PICCARD-ek soluzio bat bermatzen digu

$\int \frac{1}{y^5}dy = \int \frac{1}{4}dx$; $\frac{y^{-4}}{-4} = \frac{x}{4} + K$; $\frac{1}{y^4} = -x + K$, $K \in \mathbb{R}$

HASTAPEN BALDINTZAKO PROBLEMA APLIKATU

$\frac{1}{1^4} = 0 + K$; $\boxed{K=1}$

↓ HORTAZ

$y^4 = \dfrac{1}{1-x}$; $\boxed{y = \sqrt[4]{\dfrac{1}{1-x}}}$ $\forall x \in (-\infty, 1)$

④

A) $y' + 3y = \underbrace{x + e^{-2x}}_{q(x)}$

(1) $y' + 3y = 0$ ($L[y]=0$ EDA lineal homogeneoa)

$y' = \frac{dy}{dx} = -3y$; $\frac{dy}{y} = -3dx$; $\int \frac{dy}{y} = \int -3dx$; $L|y| = -3x + K$;

$y = e^{-3x+K}$; $\boxed{y_H = Ke^{-3x}}$ $K \in \mathbb{R}$

(2) $y_P = K(x)e^{-3x}$ NON $y'_P = K'(x)e^{-3x} - 3K(x)e^{-3x}$

↓ EDA OSOAN ORDEZKATU

$K'(x)e^{-3x} \cancel{- 3K(x)e^{-3x} + 3K(x)e^{-3x}} = x + e^{-2x}$; $K'(x) = \dfrac{x + e^{-2x}}{e^{-3x}}$;

ZATIKAKO METODOAN
↓
ALPES

$K'(x) = e^{3x}x + e^{x} \xrightarrow{\text{INTEGRATU}} K(x) = \int (e^{3x}x)dx + \int e^x dx =$

$= \left\{ \begin{array}{ll} u = x & du = 1\,dx \\ dv = e^{3x}dx & v = \frac{1}{3}e^{3x} \end{array} \right\} = \left(x\frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}dx \right) + e^x$;

$\boxed{K(x) = \dfrac{xe^{3x}}{3} - \dfrac{e^{3x}}{9} + e^x}$ $y_P = \dfrac{x}{3} - \dfrac{1}{9} + e^{-2x}$

Atal honetako aduerazpenean K(x)-en balioa ordezkatu

$\longrightarrow \boxed{y_{SO} = Ke^{-3x} + \dfrac{x}{3} - \dfrac{1}{9} + e^{-2x}}$

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B) $y' - 2y = x^2 e^{2x}$

(1) $y' - 2y = 0$ ($L[y] = 0$ EDA lineal homogeneoa)

$y' = \frac{dy}{dx} = 2y$; $\frac{dy}{y} = 2dx$; $\int \frac{dy}{y} = \int 2dx$; $L|y| = 2x + k$;

$y = e^{2x+k}$; $\boxed{y_H = ke^{2x}}$

(2) $y_P = k(x)\, e^{2x}$ NON $y_P' = k'(x)\, e^{2x} + 2k(x)\, e^{2x}$

↓ EDA OSOAN ORDEZKATU

$k'(x)\, e^{2x} + 2k(x) e^{2x} - 2k(x) e^{2x} = x^2 e^{2x}$; $k'(x) = \frac{x^2 e^{2x}}{e^{2x}}$;

$\boxed{k'(x) = x^2}$ $\xrightarrow{\text{INTEGRATU}}$ $\boxed{k(x) = \frac{x^3}{3}}$

$\left[y_P = \frac{x^3}{3} e^{2x}\right]$ $\longrightarrow$ $\boxed{y_{SO} = ke^{2x} + \frac{x^3}{3} e^{2x}}$

C) $y' + \frac{y}{x} = 3\cos(2x)$

(1) $y' + \frac{y}{x} = 0$ ($L[y] = 0$ EDA lineal homogeneoa)

$y' = \frac{dy}{dx} = -\frac{y}{x}$; $\frac{dy}{y} = -\frac{dx}{x}$; $\int \frac{dy}{y} = -\int \frac{dx}{x}$; $L|y| = -L|x| + k$;

$y = \exp\left[L|1/x| + k\right]$; $\boxed{y_H = \frac{k}{x}}$

(2) $y_P = \frac{k(x)}{x}$ NON $y_P' = \frac{k'(x)x - k(x)}{x^2}$

↓ EDA OSOAN ORDEZKATU

$\frac{k'(x)x}{x^2} - \frac{k(x)}{x^2} + \frac{k(x)/\cancel{x}}{\cancel{x}} = 3\cos(2x)$, $k'(x) = 3x\cos(2x)$

k(x) LORTZEKO INTEGRATU $\longrightarrow$ $k(x) = \int 3x\cos(2x)\,dx = \left\{\begin{matrix} u = 3x & du = 3dx \\ dv = \cos(2x)dx & v = \frac{1}{2}\sin(2x) \end{matrix}\right\} =$

$= 3x\,\frac{1}{2}\sin(2x) - \int \frac{1}{2}\sin(2x)\,3dx = \boxed{\frac{3x}{2}\sin(2x) + \frac{3}{4}\cos(2x) = k(x)}$

$y_P = \frac{\frac{3x}{2}\sin(2x) + \frac{3}{4}\cos(2x)}{x} = \frac{3}{2}\sin(2x) + \frac{3}{4x}\cos(2x)$

$$\boxed{y_{SO} = \frac{k}{x} + \frac{3}{2}\sin(2x) + \frac{3}{4x}\cos(2x)}$$

⑤

A) $\begin{cases} y' + 2y = x e^{-2x} \\ y(1) = 0 \end{cases}$

(1) $y' + 2y = 0$ ($L[y] = 0$ EDA lineal homogeneoa)

$y' = -2y$; $\frac{dy}{dx} = -2y$; $\frac{dy}{y} = -2dx$; $\int \frac{dy}{y} = -\int 2dx$;

$L|y| = -2x + K$; $y = e^{-2x+K}$; $\boxed{y_H = K e^{-2x}}$

(2) $y_P = K(x) e^{-2x}$ NON $y_P' = K'(x) e^{-2x} + K(x)(-2) e^{-2x}$

↓ EDA OSOAN aplikatu

$K'(x) e^{-2x} \cancel{-2K(x) e^{-2x} + 2K(x) e^{-2x}} = x e^{-2x}$;

$K'(x) = \frac{x e^{-2x}}{e^{-2x}}$; $\boxed{K'(x) = x}$

$K'(x)$ INTEGRATU $K(x)$ LORTZEKO

$K(x) = \int K'(x) dx = \int x \, dx = \frac{x^2}{2}$

↓ HORTAZ

$y_P = \frac{x^2}{2} e^{-2x} \longrightarrow \boxed{y_{SO} = K e^{-2x} + \frac{x^2}{2} e^{-2x}}$

HASTAPEN BALDINTZAKO PROBLEMA AINTZAT HARTUZ $(y(1) = 0)$

$0 = K e^{-2} + \frac{1}{2} e^{-2}$; $K e^{-2} = \frac{e^{-2}}{2}$; $\boxed{K = \frac{1}{2}}$

$$\boxed{y = \frac{e^{-2x}}{2} + \frac{x^2}{2} e^{-2x}}$$

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B) $\begin{cases} y' + \frac{2}{x} y = \frac{\cos x}{x^2} \\ y(\pi) = 0 \\ x > 0 \end{cases}$

(1) $y' + \frac{2}{x} y = 0$ ($L[y] = 0$ EDA lineal homogeneoa)

$y' = -\frac{2}{x} y$; $\frac{dy}{dx} = -\frac{2}{x} y$; $\frac{dy}{y} = -\frac{2}{x} dx$; $\int \frac{dy}{y} = \int \left(-\frac{2}{x}\right) dx$;

$L|y| = -2 L|x| + K$; $L|y| = L|1/x^2| + K$; $\boxed{y_H = K \cdot \frac{1}{x^2}}$

(2) $y_P = K(x) \frac{1}{x^2}$ ETA $y'_P = K'(x) \frac{1}{x^2} + K(x)(-2) \frac{1}{x}$

↓ EDA OSOAN ORDEZKATU

$K'(x) \frac{1}{x^2} - 2K(x) \frac{1}{x} + \frac{2}{x} K(x) \frac{1}{x^2} = \frac{\cos x}{x^2}$; $K'(x) \frac{1}{x^2} = \frac{\cos x}{x^2}$;

$\boxed{K'(x) = \cos x} \xrightarrow{\text{INTEGRATU}} \boxed{K(x) = \int K'(x) dx = \int \cos x \, dx = \sin x}$

$\boxed{y_P = \sin x \frac{1}{x^2}} \longrightarrow \boxed{y_{SO} = \frac{K}{x^2} + \sin x \frac{1}{x^2}}$

HASTAPEN BALDINTZAKO PROBLEMA APLIKATU $y(\pi) = 0$

$0 = \frac{K}{\pi^2} + \sin \pi \frac{1}{\pi^2}$; $\frac{K}{\pi^2} = \underbrace{\sin \pi}_{0} \frac{1}{\pi^2}$; $\boxed{K = 0}$

$\boxed{y = \sin x \frac{1}{x^2}}$

c) $\begin{cases} y' - 2y = e^{2x} \\ y(0) = 2 \end{cases}$

(1) $y' - 2y = 0$ ($L[y] = 0$ EDA lineal homogeneoa)

$y' = \frac{dy}{dx} = 2y$; $\frac{dy}{y} = 2dx$; $\int \frac{dy}{y} = 2 \int dx$; $L|y| = 2x + K$;

$y = e^{2x+K}$; $\boxed{y_H = K e^{2x}}$

(2) $y_P = K(x) e^{2x}$ ETA $y'_P = K'(x) e^{2x} + 2K(x) e^{2x}$

↓ EDA OSOAN ORDEZKATU

$K'(x) e^{2x} + 2K(x) e^{2x} - 2K e^{2x} = e^{2x}$; $K'(x) = \frac{e^{2x}}{e^{2x}}$; $\boxed{K'(x) = 1}$

• $K(x) = \int K'(x) dx = \int dx = x$

$\boxed{y_P = x e^{2x}} \longrightarrow \boxed{y_{SO} = K e^{2x} + x e^{2x}}$

HASTAPEN BALDINTZAKO PROBLEMA APLIKATUZ $(y(0)=2)$

$2 = k e^0 + 0 e^0$; $\boxed{k=2}$

$$\boxed{y = 2e^{2x} + x e^{2x}}$$

D) $\begin{cases} xy' + 2y = \sin x \\ y(\pi/2) = 1 \end{cases}$

(1) $xy' + 2y = 0$ $\quad$ ($L[y]=0$ EDA lineal homogeneoa)

$xy' = -2y$; $y' = \frac{dy}{dx} = -\frac{2y}{x}$; $\frac{dy}{y} = -2\frac{dx}{x}$; $\int \frac{dy}{y} = -2\int \frac{dx}{x}$;

$L|y| = -2L|x| + K$; $L|y| = L|1/x^2| + K$; $\boxed{y_H = \frac{1}{x^2} K}$

(2) $y_P = K(x)\frac{1}{x^2}$ $\quad$ ETA $\quad y_P' = K'(x)\frac{1}{x^2} + K(x)\left(-2\frac{1}{x^3}\right)$

↓ EDA OSOAN APLIKATU

$x\left(K'(x)\frac{1}{x^2} - 2K(x)\frac{1}{x^3}\right) + 2K(x)\frac{1}{x^2} = \sin x$;

$K'(x)\frac{1}{x} \cancel{- 2K(x)\frac{1}{x^2}} \cancel{+ 2K(x)\frac{1}{x^2}} = \sin x$; $\boxed{K'(x) = x\sin x}$

$K(x) = \int K'(x)\,dx = \int x\sin x\,dx = \begin{Bmatrix} u = x & du = dx \\ dv = \sin x\,dx & v = -\cos x \end{Bmatrix} =$

$= x\cdot(-\cos x) - \int(-\cos x)\,dx = -x\cos x + \int \cos x\,dx = \boxed{-x\cos x + \sin x}$

$$\left[y_P = \frac{-x\cos x + \sin x}{x^2}\right] \rightsquigarrow \boxed{y_{SO} = \frac{K}{x^2} + \frac{-x\cos x + \sin x}{x^2}}$$

HASTAPEN BALDINTZA PROBLEMA AINTZAT HARTUZ $y(\pi/2)=1$

$1 = \frac{K}{(\pi/2)^2} + \frac{-\pi/2\,\overbrace{\cos(\pi/2)}^{0} + \overbrace{\sin(\pi/2)}^{1}}{(\pi/2)^2}$; $\frac{\pi^2}{4} = K + 1$; $\boxed{K = \frac{\pi^2}{4} - 1}$

$$\boxed{y = \frac{\left(\frac{\pi^2}{4} - 1\right)}{x^2} + \frac{-x\cos x + \sin x}{x^2}}$$

$-\frac{3}{2}-1=-\frac{5}{2}$ $\quad (x^2+y^2)^{-3/2} \longrightarrow -\frac{3}{2}(x^2+y^2)^{-5/2}\,2y$

8.

B) $\underbrace{\frac{x}{(x^2+y^2)^{3/2}}}_{M}dx+\underbrace{\frac{y}{(x^2+y^2)^{3/2}}}_{N}dy=0$

$$\frac{\partial M}{\partial y}=x\left(-\frac{3}{2}(x^2+y^2)^{-5/2}\,2y\right)$$

$=$ EDA ZEHATZA

$$\frac{\partial N}{\partial x}=y\left(-\frac{3}{2}(x^2+y^2)^{-5/2}\,2x\right)$$

- N INTEGRATU y-rekiko $\left(\int N\,dy=\varphi \text{ SOLUZIOA}\right)$

$$\int\frac{y}{(x^2+y^2)^{3/2}}dy=\int y(x^2+y^2)^{-3/2}dy=\frac{1}{2}\int 2y(x^2+y^2)^{-3/2}dy;$$

$$\int N\,dy=\boxed{\frac{1}{2}\frac{(x^2+y^2)^{-1/2}}{-1/2}+h(x)=\varphi(x,y)}$$

↓ DERIBATU x-rekiko → φ x-ekiko deribatuz M lortuko baitugu

$$M=\frac{d\varphi}{dx}=\frac{1}{2}(x^2+y^2)^{-3/2}\cdot 2x+h'(x);\quad x(x^2+y^2)^{-3/2}+h'(x)-M=0;$$

$$h'(x)=\frac{x}{(x^2+y^2)^{3/2}}-\frac{x}{(x^2+y^2)^{3/2}};\quad \boxed{h'(x)=0}\xrightarrow{\text{ORDUAN}}\boxed{h(x)=0}$$

$$\varphi=-(x^2+y^2)^{-1/2}=K;\quad \boxed{K=\frac{1}{(x^2+y^2)^{1/2}}}$$

C) $\underbrace{\cos x\cos^2 y}_{M}\,dx+\underbrace{2\sin x\sin y\cos y}_{N}\,dy=0$

$$\frac{\partial M}{\partial y}=\cos^2 y+\cos x\cdot 2\cos y\cdot(-\sin y)$$

$\neq$ EZ DA EDA ZEHATZA

$$\frac{\partial N}{\partial x}=2\cos x\sin y\cos y+2\sin x$$

10 HOMOGENEOAK DIREZA FROGATU ⊕ EBATZI

A) $\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$; $\underbrace{(x^2+3y^2)}_{M}dx = \underbrace{2xy}_{N}dy$

M eta N 2. ORDENAKO FUNTZIO HOMOGENEOAK $\longrightarrow$ EDA HOMOGENEOA

• $y' = \frac{x^2+3y^2}{2xy} \xrightarrow[\text{2. ordenakoa delez}]{x^2\text{-gatik zatitu}} y' = \frac{1+3\left(\frac{y}{x}\right)^2}{2\,y/x} \overset{⊕}{=} \frac{1+3u^2}{2u}$

• $\overset{⊕}{u} = y/x \longrightarrow y = xu \xrightarrow{\text{DERIBATU}} y' = u'x + u$

y'-ren 2 adierazpenak berdindu

$u'x + u = \frac{1+3u^2}{2u}$; $u'x = \frac{1+3u^2}{2u} - u$; $u'x = \frac{1+3u^2-2u^2}{2u}$;

$\frac{du}{dx}x = \frac{u^2+1}{2u}$; $\frac{2u}{u^2+1}du = \frac{1}{x}dx$; $\int \frac{2u}{u^2+1}du = \int \frac{1}{x}dx$;

$L|u^2+1| = L|x| + k$; $\boxed{u^2+1 = kx}$

Aldaketa desegin $u = y/x$ IZANIK

$\left(\frac{y}{x}\right)^2 + 1 = kx$; $\frac{y^2}{x^2} + 1 = kx$; $\frac{y^2+x^2}{x^2} = kx$; $\boxed{x^2+y^2 = kx^3}$

B) $\underbrace{(3xy+y^2)}_{M}dx + \underbrace{(x^2+xy)}_{N}dy = 0$; $\frac{dy}{dx} = -\frac{3xy+y^2}{x^2+xy}$

M eta N 2. ORDENAKO FUNTZIO HOMOGENEOAK $\longrightarrow$ EDA HOMOGENEOA

• $y' = -\frac{3xy+y^2}{x^2+xy} \xrightarrow[\text{2. ORDENAKOA}]{x^2\text{-gatik ZATITU}} y' = -\frac{\frac{3y}{x}+\frac{y^2}{x^2}}{1+y/x} = -\frac{3u+u^2}{1+u}$

• $u = y/x \longrightarrow y = xu \xrightarrow{\text{DERIBATU}} y' = u'x + u$

y'-ren 2 adierazpenak berdindu

• $u'x + u = -\frac{3u+u^2}{1+u}$; $u'x = -\frac{3u+u^2}{1+u} - u$; $u'x = \frac{-3u-u^2-u-u^2}{1+u}$;

$\frac{du}{dx}x = \frac{-2u^2-4u}{1+u}$; $\frac{1+u}{-2u^2-4u}du = \frac{1}{x}dx = -\frac{1}{2}\cdot\frac{1}{2}\int\frac{1+u}{u^2+2u}\cdot 2\,du = \int\frac{1}{x}dx$;

$-\frac{1}{4}L|u^2+2u| = L|x| + k$; $L|u^2+2u| = L|1/x^4| + k$ (-4-ak ez dio eragiten egiten k-ri)

$\boxed{u^2+2u = k\frac{1}{x^4}}$

Aldaketa desegin $u = y/x$ IZANIK

$\left(\frac{y}{x}\right)^2 + 2\frac{y}{x} = k\frac{1}{x^4}$; $\boxed{y^2x^2 + 2yx^3 = k}$

⑥

A) $\begin{cases} xy' + 2y = x^2 - x + 1 \\ y(1) = 1/2 \end{cases}$

(1) $xy' + 2y = 0$ ($L[y] = 0$ EDA lineal homogeneoa)

$xy' = -2y$; $\frac{dy}{dx}x = -2y$; $\frac{1}{-2y}dy = \frac{1}{x}dx$; $\int \frac{1}{-2y}dy = \int \frac{1}{x}dx$;

$\frac{1}{-2}L|y| = L|x| + k$; $L|y| = -2L|x| + k$; $L|y| = L|1/x^2| + k$;

$$\boxed{y_H = k\frac{1}{x^2}}$$

(2) $y_P = \frac{k(x)}{x^2}$ ETA $y'_P = \frac{k'(x)x^2 - k(x)2x}{x^4}$

↓ EDA OSOAN ORDEZKATU

$x \cdot \left(\frac{k'(x)x^2 - 2x\,k(x)}{x^4}\right) + 2\frac{k(x)}{x^2} = x^2 - x + 1$;

$\frac{k'(x)}{x} - \cancel{2\frac{k(x)}{x^2}} + \cancel{2\frac{k(x)}{x^2}} = x^2 - x + 1$; $\frac{k'(x)}{x} = x^2 - x + 1$;

$k'(x) = x^3 - x^2 + x \xrightarrow{\text{INTEGRATU}} \int k'(x)dx = \int (x^3 - x^2 + x)dx$;

$$\boxed{k(x) = \frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2}}$$

$$\boxed{y_{SO} = \frac{k}{x^2} + \frac{x^2}{4} - \frac{x}{3} + \frac{1}{2}}$$

$x \in (0, \infty)$

Logaritmoaren barnean $x > 0$ izan behar baita.

HASTAPEN BALDINTZAKO PROBLEMA $\boxed{y(1) = 1/2}$

$\cancel{\frac{1}{2}} = k + \frac{1}{4} - \frac{1}{3} + \cancel{\frac{1}{2}}$; $k = -\frac{1}{4} + \frac{1}{3} = \frac{-3+4}{12}$;

$\boxed{k = 1}$

HORTAZ

$$\boxed{y = \frac{1}{12x^2} + \frac{x^2}{4} - \frac{x}{3} + \frac{1}{2}}$$

(11) BERNOUILLI

c) $x^2y' + 2xy - y^3 = 0$; $y' = -\frac{2xy}{x^2} + \frac{y^3}{x^2}$; $y' = -\frac{2y}{x} + \frac{y^{3}}{x^2}$ → $\boxed{p=3}$

$u = y^{1-p} = y^{1-3} = y^{-2} = \frac{1}{y^2}$

↓ DERIBATU

$u' = (-2)\frac{1}{y^3}y' \xrightarrow{y' \text{ BAKANDU}} \boxed{y' = -\frac{y^3 u'}{2}}$

y'-ren 2 adierazpenak BERDINDU

$-\frac{y^3u'}{2} = -\frac{2y}{x} + \frac{y^3}{x^2}$; u' BAKANDU $u' = \left(-\frac{2y}{x}\right)\cdot\left(-\frac{2}{y^3}\right) + \left(\frac{y^3}{x^2}\right)\cdot\left(-\frac{2}{y^3}\right)$;

$\boxed{u' = \frac{4}{xy^2} - \frac{2}{x^2}}$ $\xrightarrow{u = 1/y^2 \text{ dela kontuan hartuz}}$ $u' = \frac{4u}{x} - \frac{2}{x^2}$

$u' - \frac{4u}{x} = -\frac{2}{x^2}$

(1) $u' - \frac{4u}{x} = 0$ ($L[u] = 0$ EDA lineal homogeneoa)

$u' = \frac{4u}{x}$; $\frac{du}{dx} = \frac{4u}{x}$; $\frac{1}{u}du = \frac{4}{x}dx$; $\int \frac{1}{u}du = 4\int\frac{1}{x}dx$;

$L|u| = 4 L|x| + K$; $\boxed{u_H = Kx^4}$ 3/4

(2) $u_p = K(x)\, x^4$ ETA $u_p' = K'(x)x^4 + K(x)4x^3$

↓ EDA OSOAN ORDEZKATU

$K'(x)x^4 + \cancel{K(x)4x^3} - \cancel{\frac{4K(x)x^4}{x}} = -\frac{2}{x^2}$;

$K'(x)x^4 = -\frac{2}{x^2}$; $\boxed{K'(x) = -\frac{2}{x^6}}$

↓ INTEGRATU

$K(x) = \int K'(x)dx = -\int\frac{2}{x^6}dx = -2\int\frac{1}{x^6} = 10\frac{1}{x^5}$

$u_{30} = Kx^4 + 10\frac{1}{x^5}x^4$

11) BERNOULLI

y'-ren aurrean dagoena kendu behar da BETI

A) $x^2y' + xy + y^2 = 0$; $y' + \frac{1}{x}y = -\frac{1}{x^2}y^{2}$ $\to p$

BERNOULLI $\boxed{p=2}$ IZANIK

$u = y^{1-p} = y^{1-2} = 1/y$

$u' = (1/y)' = -1/y^2 \cdot y'$; $u' = -u^2 \cdot y'$; $\boxed{y' = -\frac{u'}{u^2}}$

y'-ren 2 adierazpenak berdindu

$-\frac{u'}{u^2} = -\frac{1}{x}\cdot\frac{1}{u} - \frac{1}{x^2}\cdot\frac{1}{u^2}$; $u' = \frac{u}{x} + \frac{1}{x^2}$; $u' - \frac{u}{x} = \frac{1}{x^2}$

(1) $u' - \frac{u}{x} = 0$ ($L[u] = 0$ EDA lineal homogeneoa)

$u' = \frac{u}{x}$; $\frac{du}{dx} = \frac{u}{x}$; $\frac{du}{u} = \frac{dx}{x}$; $\int \frac{du}{u} = \int \frac{dx}{x}$; $L|u| = L|x| + k$;

$\boxed{u_H = kx}$ $k \in \mathbb{R}$

(2) $u_P = k(x)\, x$ ETA $u_P' = k'(x)\, x + k(x)$

↓ EDA OSOAN ORDEZKATU

$k'(x)\, x + \cancel{k(x)} - \frac{\cancel{k(x)\, x}}{x} = \frac{1}{x^2}$; $k'(x)\, x = \frac{1}{x^2}$; $\boxed{k'(x) = \frac{1}{x^3}}$

$k'(x)$ INTEGRATUZ $\longrightarrow$ $\int k'(x)\, dx = \int \frac{1}{x^3} dx$; $\boxed{k(x) = -\frac{1}{2x^2}}$

SOLUZIOA

$\hookrightarrow$ $u_{SO} = kx - \frac{1}{2x^2}x$; $\boxed{u_{SO} = kx - \frac{1}{2x}}$

Aldaketa desegin $u = 1/y$

SOLUZIOA $\longrightarrow$ $\frac{1}{y} = kx - \frac{1}{2x}$; $\boxed{y = \frac{2x}{2kx^2 - 1}}$

7. $\begin{cases} y' + p(x)\, y = 0 \\ y(0) = 1 \end{cases}$ NON $p(x) = \begin{cases} 2 & 0 \le x \le 1 \quad (A) \\ 1 & x > 1 \end{cases}$

A $\quad y' = -2y$

$$\left.\begin{aligned} f(x,y) &= -2y \\ \frac{\partial f}{\partial y} &= -2 \end{aligned}\right]$$ JARRAITUAK DIRA $(0,1)$ PUNTUAN

PICCARD

$\frac{dy}{dx} = -2y$; $\frac{1}{y} dy = -2dx$; $\int \frac{1}{y} dy = -2 \int dx$; $L|y| = -2x + k$;

$y_1 = e^{-2x+k}$; $\boxed{y_1 = k e^{-2x}}$

HASTAPEN BALDINTZA APLIKATUZ $y(0) = 1$

$1 = k e^{-2 \cdot 0}$; $\boxed{k = 1}$ $\longrightarrow$ $\boxed{y_1 = e^{-2x}}$

B $\quad y' = -y$

$$\left.\begin{aligned} f(x,y) &= -y \\ \frac{\partial f}{\partial y} &= -1 \end{aligned}\right]$$ JARRAITUAK DIRA $(0,1)$ PUNTUAN

PICCARD

$\frac{dy}{dx} = -y$; $\frac{1}{y} dy = -dx$; $\int \frac{1}{y} dy = -\int dx$; $L|y| = -x + k$;

$y_2 = e^{-x+k}$; $\boxed{y_2 = k e^{-x}}$

HASTAPEN BALDINTZA APLIKATUZ

$1 = k e^{-0}$; $\boxed{k = 1}$ $\longrightarrow$ $\boxed{y_2 = e^{-x}}$

6.

c) $\begin{cases} xy' + 2y = \sin x \\ y(\pi) = 1/\pi \end{cases}$

(1) $xy' + 2y = 0$ ($L[y]=0$ EDA lineal homogeneoa)

$xy' = -2y$; $x\frac{dy}{dx} = -2y$; $\frac{1}{y}dy = -2\frac{1}{x}dx$; $\int \frac{1}{y}dy = -2\int \frac{1}{x}dx$;

$L|y| = -2L|x| + k$; $L|y| = L|1/x^2| + k$; $\boxed{y_H = k\frac{1}{x^2}}$

(2) $y_P = k(x)\frac{1}{x^2}$ ETA $y_P' = k'(x)\frac{1}{x^2} + k(x)(-2)\frac{1}{x^3}$

↓ EDA OSOAN ORDEZKATU

$x\left(k'(x)\frac{1}{x^2} - 2k(x)\frac{1}{x^3}\right) + 2k(x)\frac{1}{x^2} = \sin x$;

$\frac{k'(x)}{x} - \cancel{\frac{2k(x)}{x^2}} + \cancel{\frac{2k(x)}{x^2}} = \sin x$; $k'(x)\frac{1}{x} = \sin x$;

$[k'(x) = x \cdot \sin x] \xrightarrow{\text{INTEGRATU}} \int k'(x)dx = \int x\sin x\,dx = \left\{\begin{matrix} u = x & du = dx \\ dv = \sin x\,dx & v = -\cos x \end{matrix}\right\} =$

$= x\cdot(-\cos x) + \int \cos x\,dx = -x\cos x + \sin x$

$$\left[y_{SO} = k\frac{1}{x^2} - \frac{\cos x}{x} + \frac{\sin x}{x^2}\right]$$

HASTAPEN BALDINTZAKO PROBLEMA APLIKATUZ $y(\pi) = 1/\pi$

$\frac{1}{\pi} = \frac{k}{\pi^2} - \frac{\overbrace{\cos\pi}^{-1}}{\pi} + \cancel{\frac{\sin\pi}{\pi^2}}^{0}$; $\cancel{\frac{1}{\pi}} = \frac{k}{\pi} + \cancel{\frac{1}{\pi}}$; $\boxed{k = 0}$

$$\boxed{y = \frac{\sin x}{x^2} - \frac{\cos x}{x}} \quad \forall x \in \mathbb{R} - \{0\}$$

④

D) $y' + (\tan x) y = x \sin(2x) \qquad x \in (-\pi/2, \pi/2)$

(1) $y' + (\tan x) y = 0 \quad (L[y]=0$ EDA lineal homogeneoa)

$y' = -(\tan x) y \; ; \; \frac{dy}{dx} = -(\tan x) y \; ; \; \frac{1}{y} dy = -(\tan x) dx \; ;$

$\int \frac{1}{y} dy = -\int \tan x \, dx \; ; \; L|y| = -(-L|\cos x|) + K \; ; \; L|y| = L|\cos x| + K \; ;$

$\boxed{y_H = K \cos x}$

$\tan x = \frac{\sin x}{\cos x}$

$\sin x = \tan x \cdot \cos x$

(2) $y_P = K(x) \cos x$ ETA $y'_P = K'(x) \cos x + K(x)(-\sin x)$

↓ EDA OSOAN ORDEZKATU $\tan x$

$K'(x) \cos x - K(x) \sin x + \frac{\sin x}{\cos x} K(x) \cos x = x \sin(2x) \; ;$

$K'(x) \cos x - \cancel{K(x) \sin x} + \cancel{K(x) \sin x} = x \sin(2x) \; ; \; \boxed{K'(x) = x \frac{\sin(2x)}{\cos x}}$

$K'(x) = x \frac{2 \sin x \cos x}{\cos x} = 2x \sin x$

OHARRA!

$\sin(2x) = 2 \sin x \cdot \cos x$

↓ INTEGRATU

$K(x) = \int K'(x) dx = \int 2x \sin dx = \left\{ \begin{array}{ll} u = 2x & du = 2 dx \\ dv = \sin x dv & v = -\cos x \end{array} \right\} =$

$= 2x(-\cos x) - \int (-\cos x) \cdot 2 dx = -2x \cos x + 2 \int \cos x \, dx = \boxed{-2x \cos x + 2 \sin x = K(x)}$

$\boxed{y_{SO} = K \cos x - 2x \cos^2 x + 2 \sin x \cos x}$

⑥

B) $\begin{cases} y' + y \cot x = 2 \operatorname{cosec} x \\ y(\pi/2) = 1 \end{cases}$

OHARRA!

$\cot x = \frac{\cos x}{\sin x}$

$\operatorname{cosec} x = 1/\sin x$

(1) $y' + y \cot x = 0 \quad (L[y] = 0$ EDA lineal homogeneoa)

$y' = -y \frac{\cos x}{\sin x} \; ; \; \frac{dy}{dx} = -y \frac{\cos x}{\sin x} \; ; \; \frac{1}{y} dy = \frac{\cos x}{\sin x} dx \; ; \; \int \frac{1}{y} dy = -\int \frac{\cos x}{\sin x} dx \; ;$

$L|y| = -L|\sin x| + K \; ; \; L|y| = L|1/\sin x| + K \; ; \; \boxed{y_H = \frac{K}{\sin x}}$

(2) $y_P = \frac{K(x)}{\sin x}$ ETA $y'_P = \frac{K'(x) \sin x - K(x) \cos x}{\sin^2 x}$

↓ EDA OSOAN ORDEZKATU

$\frac{K'(x) \sin x - K(x) \cos x}{\sin^2 x} + \frac{K(x)}{\sin x} \frac{\cos x}{\sin x} = \frac{2}{\sin x} \; ; \; \frac{K'(x)}{\sin x} = \frac{2}{\sin x} \; ;$

$\boxed{K'(x) = 2} \xrightarrow{\text{INTEGRATU}} K(x) = \int K'(x) dx = \int 2 dx = 2x$

$\boxed{y_{SO} = \frac{K}{\sin x} + \frac{2x}{\sin x}}$

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↓ HASTAPEN BALDINTZAKO PROBLEMA APLIKATU

$$1 = \frac{k}{\underbrace{\sin(\pi/2)}_{1}} + \frac{2\,\pi/2}{\underbrace{\sin(\pi/2)}_{1}} \; ; \; \boxed{k = 1 - \pi}$$

$$\boxed{y_{SO} = \frac{1-\pi}{\sin x} + \frac{2x}{\sin x}}$$

$\forall x \in (0, \pi)$

π-tik aurrera (hau da, 3. eta 4. koadranteetan) $\sin x$-ak balio negatiboa edukiko du, eta logaritmoa aplikatu dugunez EZIN DUGU KONTUTAN HARTU

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10.

c) $xy' = y + xe^{y/x} \longrightarrow y' = \frac{y}{x} + e^{y/x} = u + e^{u}$

$u = y/x$ IZANIK $\xrightarrow{y = xu} y' = u + u'x$

y'-ren 2 adierazpenak berdindu

$u + e^{u} = u + u'x$; $u'x = e^{u}$; $\frac{du}{dx} x = e^{u}$; $\frac{1}{e^{u}} du = \frac{1}{x} dx$;

$\int \frac{1}{e^{u}} du = \int \frac{1}{x} dx$; $-e^{-u} = L|x| + K$; $K = -e^{-u} - L|x|$; K edozein konstante denez

$\boxed{K = e^{-u} + L|x|} \xrightarrow[\text{ALDAKETA}]{\text{ALDAGAI}} \boxed{K = e^{-y/x} + L|x|}$

D) $\frac{dy}{dx} = \frac{2y + \sqrt{x^2 - y^2}}{2x} \longrightarrow y' = \frac{2u + \sqrt{1 - u^2}}{2}$

$u = y/x$ IZANIK $\xrightarrow{y = xu} y' = u + u'x$

y'-ren 2 adierazpenak berdindu

$\frac{2u + \sqrt{1-u^2}}{2} = u + u'x$; $\sqrt{1-u^2} = \cancel{2u} + 2u'x \cancel{-2u}$;

$\sqrt{1-u^2} = 2x\frac{du}{dx}$; $\frac{1}{x} dx = \frac{2}{\sqrt{1-u^2}} du$;

$\int \frac{1}{x} dx = \int \frac{2}{\sqrt{1-u^2}} du$; $L|x| = \arcsin(u) + k$;

$\boxed{K = L|x| - \arcsin(u)} \xrightarrow[\text{ALDAKETA}]{\text{ALDAGAI}} \boxed{K = L|x| - \arcsin(y/x)}$

8.

A) $y' = -\dfrac{ax-by}{bx-cy}$

$\dfrac{dy}{dx} = -\dfrac{ax-by}{bx-cy}$; $\underbrace{(ax-by)}_{M}dx + \underbrace{(bx-cy)}_{N}dy = 0$

$\dfrac{\partial M}{\partial y} = -b$

$\dfrac{\partial N}{\partial x} = b$

$\neq$ EZ DA EDA ZEHATZA

11. BERNOULLI

D) $xy^2y' + y^3 = x\cos x \xrightarrow[\text{y' LIBRATZEKO}]{\text{ZATI } xy^2} y' + \dfrac{y}{x} = \dfrac{\cos x}{y^{2}}$ → $\boxed{p=-2}$ BERNOULLI

$u = y^{1-p} = y^{1-(-2)} = y^3$

↓ DERIBATU

$u' = 3y^2y' \xrightarrow{\text{y' BAKANDU}} y' = \dfrac{u'}{3y^2}$

y'-ren 2 adierazpenak berdindu

$\dfrac{u'}{3y^2} = \dfrac{\cos x}{y^2} - \dfrac{y}{x}$; $u' = \dfrac{3\cos x\, \cancel{y^2}}{\cancel{y^2}} - \dfrac{3y^3}{x}$; $\boxed{u' = 3\cos x - \dfrac{3u}{x}}$

$u' + \dfrac{3u}{x} = 3\cos x$

(1) $u' + \dfrac{3u}{x} = 0$ ($L[u]=0$ EDA lineal homogeneoa)

$u' = -\dfrac{3u}{x}$; $\dfrac{du}{dx} = -\dfrac{3u}{x}$; $-\dfrac{1}{x}dx = \dfrac{1}{3u}du$; $-\int\dfrac{1}{x}dx = \int\dfrac{1}{3u}du$;

$-L|x| = \dfrac{1}{3}L|u| + K$; $L|u| = L|1/x^3| + K$; $\boxed{u_H = \dfrac{K}{x^3}}$

(2) $u_P = \dfrac{K(x)}{x^3}$ ETA $u_P' = \dfrac{K'(x)x^3 - K(x)(3x^2)}{x^6}$

↓ EDA OSOAN APLIKATU

$\dfrac{K'(x)x^3}{x^6} - \cancel{\dfrac{3x^2K(x)}{x^6}} + \cancel{\dfrac{3K(x)}{x^3}\cdot\dfrac{1}{x}} = 3\cos x$; $\dfrac{K'(x)}{x^3} = 3\cos x$

$\boxed{K'(x) = 3x^3\cos x} \xrightarrow[\text{K(x) lortzeko}]{\text{INTEGRATU}} K(x) = \int K'(x)dx = \int 3x^3\cos x\, dx =$

$\left\{\begin{matrix} u = 3x^3 & du = 9x^2dx \\ dv = \cos x dx & v = \sin x \end{matrix}\right\} = 3x^3\sin x - \int \sin x\,(9x^2)dx = \left\{\begin{matrix} u = 9x^2 & du = 18x dx \\ dv = \sin x dx & v = -\cos x \end{matrix}\right.$

$= 3x^3\sin x - \left(9x^2(-\cos x) - \int(-\cos x)18x\,dx\right) = \left\{\begin{matrix} u = 18x & du = 18dx \\ dv = \cos x & v = \sin x \end{matrix}\right\} =$

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$$= 3x^3\sin x - \left(-\cos x \cdot 9x^2 + \left(18x\cdot\sin x - \int \sin x \cdot 18\,dx\right)\right) =$$

$$= 3x^3\sin x - \left(-\cos x\cdot 9x^2 - \left(18x\sin x - \left((-\cos x)\,18\right)\right)\right) =$$

$$= 3x^3\sin x + 9x^2\cos x - 18x\sin x + 18\cos x$$

$$\boxed{y_{SO} = 3\sin x + \frac{9\cos x}{x} - \frac{18\sin x}{x^2} + \frac{18\cos x}{x^3} + \frac{K}{x^3}}$$

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