ARIKETAK

3. $p(x) = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5$

TAYLOR GARAPEN INFINITUA

↳ Polinomioaren Taylor garapena kalkulatu $(x-1)$ berreduretan HÖRNER erabiliz

NON $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

↓ HAU DA

$$\sum_{n=0}^{5} a_n (x-1)^n = \sum_{n=0}^{5} \frac{p^{n)}(1)}{n!}(x-1)^n$$

Aurreko azalpeneko adierazpena kontuan hartuz:

$$p^{k)}(r) = k!\, q_{n-2}(r); \quad \frac{p^{k)}(r)}{k!} = q_{n-2}(r)$$

HÖRNER → Ariketa honetan $r = 1$ izango da, enuntziatuak $(x-1)$-en berreduretan adieraztea eskatzen baitu.

	-6	5	-4	3	-2	1	
1		-6	-1	-5	-2	-4	
	-6	-1	-5	-2	-4	$\boxed{-3}$	$= p(1)/0!$ NON $0! = 1$
1		-6	-7	-12	-14		
	-6	-7	-12	-14	$\boxed{-18}$ $= p'(1)/1!$		
1		-6	-13	-25			
	-6	-13	-25	$\boxed{-39}$ $= p''(1)/2! = q_3(1)$			
1		-6	-19				
	-6	-19	$\boxed{-44}$ $= p'''(1)/3! = q_2(1)$				
1		-6					
	-6	$\boxed{-25}$ $= p^{iv)}(1)/4! = q_1(1)$					

↳ -6: $p^{v)}(1)/5!$

$$p(x) = -3 - 18(x-1) - 39(x-1)^2 - 44(x-1)^3 - 25(x-1)^4 - 6(x-1)^5$$

ARIKETA

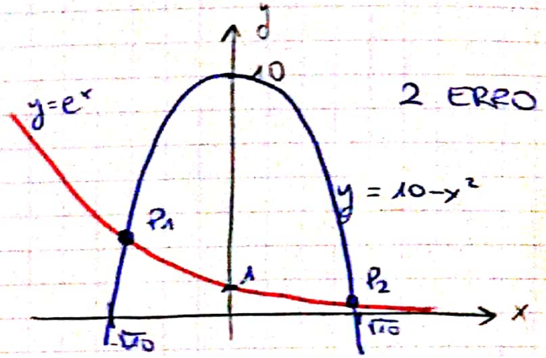
$f(x) = e^{-x} + x^2 - 10$

A) Bornatu erro guztiak grafikoki 1 luzerako tarteetan

B) Murriztu erro handienaren tartea Bisekzioaren metodoa erabiliz (P_2)

C) Burutu Regula Falsi metodoko iterazio bat p_1-en tartea murrizteko

A) $e^{-x} = 10 - x^2$



2 ERRO

$\sqrt{10}$ 3-tik gertu dagoela jakinik:

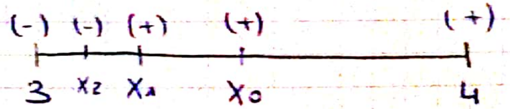
f(x)-en ordezkatuz $\begin{cases} f(3) = e^{-3}+9-10 = e^{-3}-1 < 0 \\ f(4) = e^{-4}+16-10 = e^{-4}+6 > 0 \end{cases}$

f jarraitua $P_2 \in [3,4]$ eta $f(P_2) = 0$

f(x)-en ordezkatuz $\begin{cases} f(0) = 1-10 < 0 \ (-) \\ f(-1) = e-9 < 0 \ (-) \\ f(-2) = e^2-6 > 0 \ (+) \end{cases}$ $\xrightarrow{\text{BOLTZANOREN TEOREMA}}$ $\exists p_1 \in [-2,-1]$ eta $f(p_1) = 0$

ISEKZIOAREN
TODOA

B) $p_2 = p \in [3,4] = [a_0, b_0]$



(0) $\begin{cases} x_0 = \dfrac{a_0+b_0}{2} = 3{,}5 \\ f(x_0) = 2{,}28 > 0 \end{cases}$ $\longrightarrow$ $\exists p \in [\overset{a_1}{3}, \overset{b_1}{3{,}5}]$

luz $= b_1 - a_1 = \boxed{0{,}5 > \varepsilon}$

JARRAITU !!!

(1) $\begin{cases} x_1 = \dfrac{a_1+b_1}{2} = 3{,}25 \\ f(x_1) = 0{,}601\ldots > 0 \end{cases} \longrightarrow \exists p \in [\overset{a_2}{3}; \overset{b_2}{3{,}25}]$

$lu_1 = b_2 - a_2 = 0{,}25 > \varepsilon$ JARRAITU!!!

(2) $\begin{cases} x_2 = \dfrac{a_2+b_2}{2} = 3{,}125 \\ f(x_2) = -0{,}19\ldots < 0 \end{cases} \longrightarrow \exists p \in [\overset{a_3}{3{,}125}; \overset{b_3}{3{,}25}]$

$lu_1 = b_3 - a_3 = \boxed{0{,}125 < \varepsilon}$ ETEN!!!

c) $p_1 \in [-2, -1]$

$$x_0 = \frac{a_0 f(b_0) - b_0 f(a_0)}{f(b_0) - f(a_0)}$$

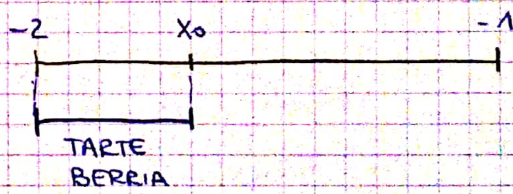
$$\hookrightarrow x_0 = \frac{2 \cdot (6{,}281719) + 1{,}381719}{-6{,}281719 - 1{,}381719} = -1{,}818916$$

$f(x_0) = -0{,}5263\ldots < 0$

$p_1 \in [-2, -1{,}818916]$

Regula Falsi metodoa dela jakinda, mutur bat geldi geratuko da eta bestea izango da mugitzen joango dena. Kasu honetan, mugitzen joango den mutura eskumakoa izango da.

HAU DA, ezkerreko mutura -2-an geratuko da geldi

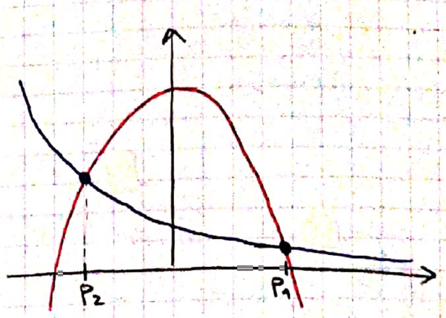


ADIB.

Aurreko ariketa batean bisekzio metodoarekin lortutakoa

$f(x) = e^{-x} + x^2 - 10$

Puntu finkoko metodo lineal baten bidez, kalkulatu P_2 7 zifra esanguratsuz



$P_2 \in (3{,}125;\ 3{,}1875)$

- g TOPATU $\longrightarrow$ ekuazioa planteatu eta x askatu

① $e^{-x} + x^2 - 10 = 0;\ e^{-x} = 10 - x^2;\ \log(e^{-x}) = \log(10 - x^2);$

$x = -\log(10 - x^2) \longrightarrow \boxed{g_1(x) = -\log(10 - x^2)}$

⊕ Balio positiboa hartuko dugu, P_2 ardatzaren alde positiboan dagoelako

② $x^2 = 10 - e^{-x};\quad x = \oplus\sqrt{10 - e^{-x}} \longrightarrow \boxed{g_2(x) = \sqrt{10 - e^{-x}}}$

$|g'(P)| < 1$? $\longrightarrow$ KONBERGITZEKO BEHARREZKOA

① $g_1(x)$ AZTERTUZ

$g_1'(x) = \dfrac{2x}{10 - x^2}$ jarraitua $\forall x \neq \pm\sqrt{10}$

g'_1-n ordezkatuko bagenu, puntu horietan jarraitua ez dela ikusiko genuke.

EGIAZTATZEKO "p_2-ren ezker muturra" ordezkatuko dugu.

$g_1'(3{,}125) = 26{,}66 \longrightarrow$ EZ DA 1 BAINO TXIKIAGOA

$\hookrightarrow$ Ez du $|g_1'(x)| < 1$ baldintza betetzen

② $g_2(x)$ AZTERTUZ

$g_2'(x) = \dfrac{e^{-x}}{2\sqrt{10 - e^{-x}}}$

EGIAZTATZEKO "P_2-ren ezker muturra" ordezkatuko dugu

$g_2'(3{,}125) = 0{,}006962$

$g_2'(3{,}1875) = 0{,}006592$ ← P_2-ren "eskuin muturra"

g_2' JARRAITUA

$\rightarrow g_2'(P) \simeq 0{,}0068 < 1$ BALDINTZA BETE!!!

NON $X_{n+1} = g_2(X_n)$

$X_0 = 3{,}125$ (P_2-ren ezker muturra)

$X_1 = g_2(X_0) = 3{,}155323$

$X_2 = g_2(X_1) = 3{,}155309$

$X_3 = g_2(X_2) = 3{,}155532$

$X_4 = g_2(X_3) = 3{,}155532$

= ETEN!!! Berdinak direnez, metodoak konbergituko du.

HORTAZ, $\boxed{P_2 \simeq 3{,}155532 = X_4}$

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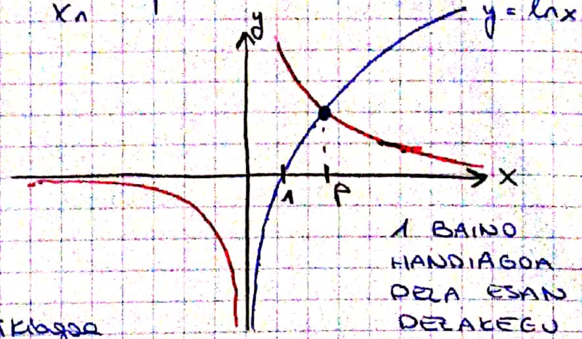
(3) PUNTU FINKOKO METODOA

$5x \cdot \ln x - 1 = 0$ { 6 digitu ~~zuzenak~~ ; $\%0{,}1 \longrightarrow \left|\frac{x_n - x_{n-1}}{x_n}\right| \cdot 100 < 0{,}1$

$\%0{,}1 \downarrow$ ZEHAZTASUNA

$\ln x = \frac{1}{5x}$

$f(x) = 5x \ln x - 1$



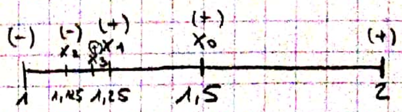
1 BAINO HANDIAGOA DELA ESAN DEZAKEGU

- $f(1) = -1 < 0$
- $f(2) = 10 \ln 2 - 1 > 0$
- f jarraitua $(1,2)$ tartean

BOLZANO APLIKATU

0,1 edo txikiagoa den tarte berri bat lortzeko

- BOLZANO $\exists p \in [1,2] \quad f(p) = 0$



(0) $\begin{cases} x_0 = \frac{1+2}{2} = 1{,}5 \\ f(x_0) = 2{,}04 \ldots > 0 \end{cases} \longrightarrow \exists p \in [1; 1{,}5]$

$l_{u7} = 1{,}5 - 1 = 0{,}5 > 0{,}1$ JARRAITU!

(1) $\begin{cases} x_1 = \frac{1+1{,}5}{2} = 1{,}25 \\ f(x_1) = 0{,}39 \ldots > 0 \end{cases} \longrightarrow \exists p \in [1; 1{,}25]$

$l_{u7} = 1{,}25 - 1 = 0{,}25 > 0{,}1$ JARRAITU!

(2) $\begin{cases} x_2 = \frac{1+1{,}25}{2} = 1{,}125 \\ f(x_2) = -0{,}337 \ldots < 0 \end{cases} \longrightarrow \exists p \in [1{,}125; 1{,}25]$

$l_{u7} = 1{,}25 - 1{,}125 = 0{,}125 > 0{,}1$ JARRAITU!

(3) $\begin{cases} x_3 = \frac{1{,}125 + 1{,}25}{2} = 1{,}1875 \\ f(x_3) = 0{,}0203 \ldots > 0 \end{cases} \longrightarrow \exists p \in [1{,}125; 1{,}1875]$

ETEN!

$l_{u7} = 1{,}1875 - 1{,}125 = 0{,}0625 < 0{,}1$

HORTAZ $\longrightarrow p \in [1{,}125; 1{,}1875] \longrightarrow \{ p \approx x_0 = \frac{a+b}{2} = 1{,}15625$

- PUNTU FINKOKO METODOA

g bilatu x bakandut

① $\left[x = \frac{1}{5 \ln x} = g_1(x) \right]$

$\downarrow$ EGIAZTATZEKO $|g_1'(x)| < 1$

$g_1'(x) = \frac{1}{5} \cdot \frac{1/x}{(\ln x)^2}$

Hasierako tartearen erdiko puntua ordezkatu

$\hookrightarrow g_1'(p) \approx g_1'(1{,}15625) = |-8{,}2064| > 1$ $\boxed{\text{EZ DU BALIO}}$ (Ez du konbergitzen)

② $\ln x = \frac{1}{5x}$; $e^{\ln x} = e^{1/5x}$; $\left[x = e^{1/5x} = g_2(x) \right]$

$\downarrow$ EGIAZTATZEKO $|g_2'(x)| < 1$

$g_2'(x) = \frac{1}{5x^2} \cdot e^{1/5x}$

$\hookrightarrow g_2'(p) \approx g_2'(1{,}15625) = |-0{,}177847| < 1$ $\boxed{\text{BALIO DU}}$

Gelditzeko irizpidea bete ?

$X_0 = 1{,}15625$

$X_1 = g_2(x_0) = 1{,}18883$

$X_2 = g_2(x_1) = 1{,}18321$

$X_3 = g_2(x_2) = 1{,}18416$

① $e_1 = \left| \frac{x_1 - x_0}{x_1} \right| \cdot 100 = \% 2{,}74 > \% 0{,}01$ JARRAITU!

② $e_2 = \left| \frac{x_2 - x_1}{x_2} \right| \cdot 100 = \% 0{,}475 > \% 0{,}01$ JARRAITU!

③ $e_3 = \left| \frac{x_3 - x_2}{x_3} \right| \cdot 100 = \% 0{,}0799 < \% 0{,}01 = \varepsilon$ $\boxed{\text{ETEN !!!}}$

HORTAZ, $\boxed{p \approx x_3 = 1{,}18416}$

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ARIKETAK

1. $f(x)=\frac{e^x-1-x}{x^2}$ t=6 digitu esanguratsuz $f(0,01)$?

A) ZUZENEAN APLIKATU

$e^{0,01}=1,01005$

$e^{0,01}-1=0,01005$

$e^{0,01}-1-0,01=0,00005$

$(0,01)^2=0,0001$

$f(0,01)=\frac{0,00005}{0,0001}=0,5$

B) MACLAURIN (2. MAILA)

$e^x=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\frac{x^4}{4!}+\dots$

$$f(x)=\frac{(\cancel{1}+x+\frac{x^2}{2}+\frac{x^3}{3!}+\dots)-1-x}{x^2}=\frac{x^2/2+x^3/3!+\dots}{x^2}=\frac{\cancel{x^2}\cdot(1/2!+x/3!+x^2/4!+\dots)}{\cancel{x^2}}=$$

$$=\frac{1}{2}+\frac{x}{3!}+\frac{x^2}{4!}+\dots=\boxed{\frac{1}{2}+\frac{x}{6}+\frac{x^2}{24}}$$ Bigarren mailakoa dela kontuan hartuta

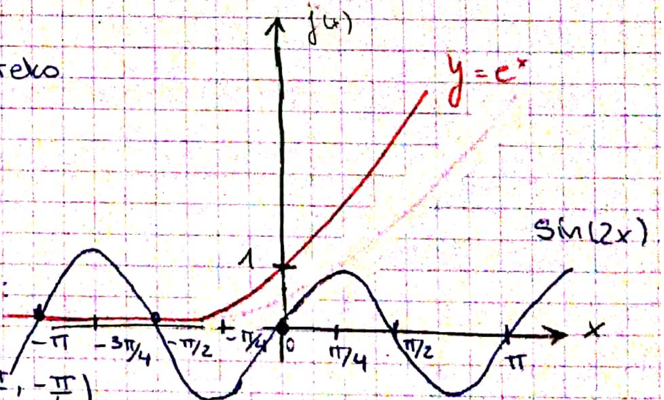
$f(0,01)=\frac{1}{2}+\frac{0,01}{6}+\frac{(0,01)^2}{24}=0,501671$ (6 digitu esanguratsuekin lan eginez prozesuan zehar)

4. $f(x)=\frac{\sin(2x)}{e^x}-1=0$ borna itzazu tarte disjuntuetan.

$f(x)$ eraldatu irudikatu ahal izateko

$\frac{\sin(2x)}{e^x}=1$; $\sin(2x)=e^x$

Soluzioak tarte hauen barnean egongo dira:



SOLUZIOAK $\in \left(-\frac{7\pi}{4},-\frac{5\pi}{4}\right),\left(-\frac{5\pi}{4},-\frac{3\pi}{4}\right),\left(-\frac{3\pi}{4},-\frac{\pi}{4}\right)$

$$\boxed{\left(-\frac{(2k+1)\pi}{4},-\frac{(2k-1)\pi}{4}\right)\quad k\geq 1}$$

TARTEAREN ADIERAZPEN OROKORRA

OHARRA!!!

$\sin(\pi/2)=1$

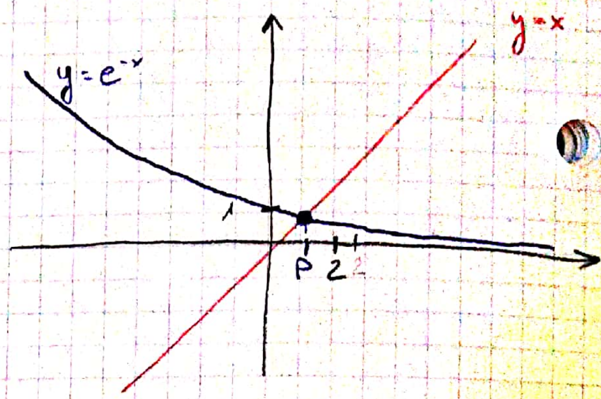
$\sin(\pi)=0$

BAINA, $\sin(2x)$ denez

$\sin(2\cdot\pi/2)=\sin(\pi)=0$

5. BISEKZIO METODOA (BOLZANO)

$\varepsilon = \% 25 = 0{,}25$ (ZEHAZTASUNA)



$f(x) = e^{-x} - x$

↓ ERALDATU

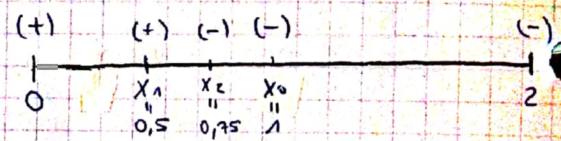
$e^{-x} - x = 0 \; ; \; \boxed{e^{-x} = x}$

Aukeratutako tartea $\longrightarrow$ $p \in [0, 2]$

$f(0) = e^{0} - 0 = 1 > 0$

$f(2) = e^{-2} - 2 < 0$ BISEKZIOA APLIKA DEZAKEGU

f jarraitua



(0) $\begin{cases} x_0 = \dfrac{a_0 + b_0}{2} = 1 \\ f(x_0) = -0{,}632\ldots < 0 \end{cases}$

(1) $\begin{cases} x_1 = \dfrac{a_1 + b_1}{2} = 0{,}5 \\ f(x_1) = 0{,}1065\ldots > 0 \end{cases}$ $\longrightarrow$ $\exists p \; [0{,}5; 1]$

$l_{WZ} = 1 - 0{,}5 = 0{,}5$ JARRAITU

(2) $\begin{cases} x_2 = \dfrac{a_2 + b_2}{2} = 0{,}75 \\ f(x_2) = -0{,}2776\ldots < 0 \end{cases}$ $\longrightarrow$ $\exists p \; [0{,}5; 0{,}75]$

$l_{WZ} = 0{,}75 - 0{,}5 = 0{,}25$

$e_{er} = \dfrac{0{,}75 - 0{,}625}{0{,}625} = \dfrac{0{,}125}{0{,}625} = 0{,}2 < \boxed{0{,}25 = \varepsilon}$ ETEN !!!

↳ Gure tartearen erdiko puntua

$\boxed{p \approx x_3 = 0{,}625}$

ZEHAZTASUNA $\longrightarrow$ ERRORE ERLATIBOA EHUNEKOTAN

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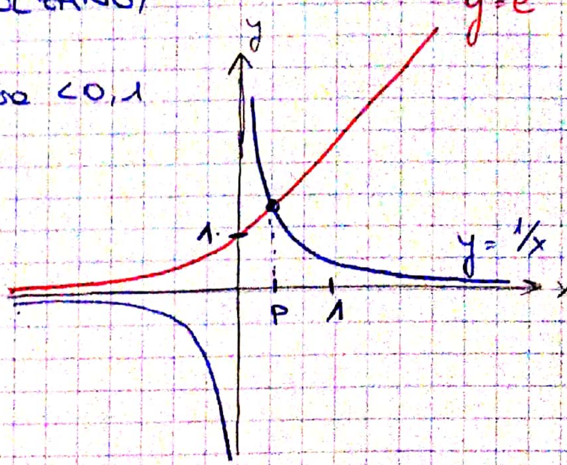
6 BISEKZIO METODOA (BOLZANO)

$f(x) = x \cdot e^x - 1$ Errorea $< 0,1$

↓ FUNTZIOA ERALDATU

$x e^x = 1$; $\boxed{e^x = 1/x}$



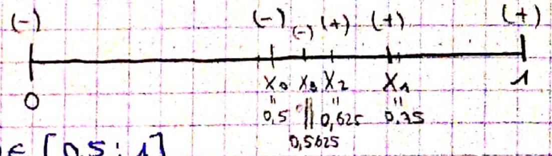
Aukeratutako tartea → $p \in [0,1]$

$f(0) = 0 \cdot e^0 - 1 = -1 < 0$

$f(1) = 1 \cdot e^1 - 1 > 0$

f jarraitua

} BISEKZIO METODOA APLIKA DAITEKE



(0) $\begin{cases} x_0 = \dfrac{a_0 + b_0}{2} = 0,5 \\ f(x_0) = -0,175 < 0 \end{cases}$ → $\begin{cases} p \in [0,5;\ 1] \\ Err = 1 - 0,5 = 0,5 > 0,1 \end{cases}$ JARRAITU!

(1) $\begin{cases} x_1 = \dfrac{a_1 + b_1}{2} = 0,75 \\ f(x_1) = 0,587\ldots > 0 \end{cases}$ → $\begin{cases} p \in [0,5;\ 0,75] \\ Err = 0,75 - 0,5 = 0,25 > 0,1 \end{cases}$ JARRAITU!

(2) $\begin{cases} x_2 = \dfrac{a_2 + b_2}{2} = 0,625 \\ f(x_2) = 0,1676\ldots > 0 \end{cases}$ → $\begin{cases} p \in [0,5;\ 0,625] \\ Err = 0,625 - 0,5 = 0,125 > 0,1 \end{cases}$ JARRAITU!

(3) $\begin{cases} x_3 = \dfrac{a_3 + b_3}{2} = 0,5625 \\ f(x_3) = -0,012\ldots < 0 \end{cases}$ → $\begin{cases} p \in [0,5625;\ 0,625] \\ Err = 0,625 - 0,5625 = 0,0625 < 0,1 \end{cases}$

ETEN !!!

$\boxed{p \in [0,5625;\ 0,625]}$

↳ $\boxed{p \approx x_4 = 0,59375}$

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7. BISEKZIO METODOA (BOLZANO)

$\sqrt[3]{3}$ balioa 0,1 baino txikiagoa den tarte batean kokatu

↓ EKUAZIO BAT LORTU

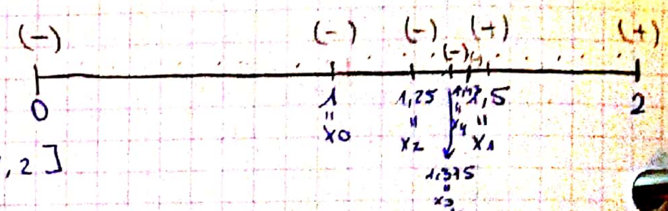
$x = \sqrt[3]{3}$, $\boxed{x^3 = 3}$

$f(x) = x^3 - 3$

$\exists p$ $[0, 2]$ tartearekin hasi → BADAKIGULAKO $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$

$f(0) = -3 < 0$
$f(2) = 5 > 0$
f jarraitua

} BISEKZIO METODOA APLIKA DAITEKE



(0) $\begin{cases} x_0 = \dfrac{a_0 + b_0}{2} = 1 \\ f(x_0) = -2 < 0 \end{cases}$ ⟶ $p \in [1, 2]$
luz $= 2 - 1 = 1 > 0{,}1$ JARRAITU!!!

(1) $\begin{cases} x_1 = \dfrac{a_1 + b_1}{2} = 1{,}5 \\ f(x_1) = 0{,}375 > 0 \end{cases}$ ⟶ $p \in [1; 1{,}5]$
luz $= 1{,}5 - 1 = 0{,}5 > 0{,}1$ JARRAITU!

(2) $\begin{cases} x_2 = \dfrac{a_2 + b_2}{2} = 1{,}25 \\ f(x_2) = -1{,}0468 < 0 \end{cases}$ ⟶ $p \in [1{,}25; 1{,}5]$
luz $= 1{,}5 - 1{,}25 = 0{,}25 > 0{,}1$ JARRAITU!

(3) $\begin{cases} x_3 = \dfrac{a_3 + b_3}{2} = 1{,}375 \\ f(x_3) = -0{,}4003\ldots < 0 \end{cases}$ ⟶ $p \in [1{,}375; 1{,}5]$
luz $= 1{,}5 - 1{,}375 = 0{,}125 > 0{,}1$ JARRAITU!

(4) $\begin{cases} x_4 = \dfrac{a_4 + b_4}{2} = 1{,}4375 \\ f(x_4) = -0{,}029\ldots < 0 \end{cases}$ ⟶ $p \in [1{,}4375; 1{,}5]$
luz $= 1{,}5 - 1{,}4375 = 0{,}0735 < 0{,}1$

ETEN!!!

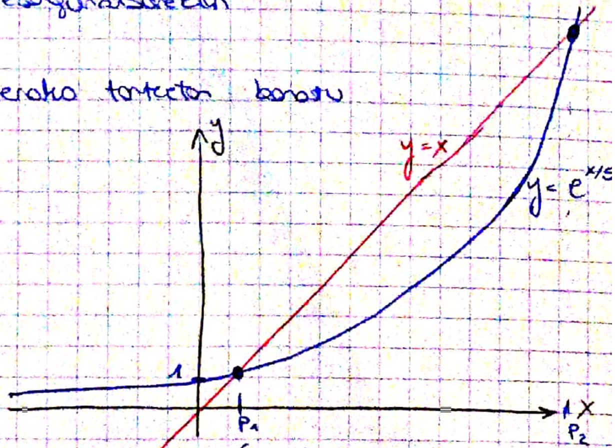
$p = \sqrt[3]{3}$ IZANIK

$\boxed{p \approx x_5 = 1{,}46875}$

HORTAZ, $\sqrt[3]{3} \approx 1{,}46875$

⑭ $f(x) = x - e^{x/5}$ $\quad t = 6$ zifra esanguratsurekin

A) Zenbat erro? Erroak: 1 luzerako tarteetan banatu



Grafikoki 2 soluzio dituela ikus daiteke

EKUAZIOA ERALDATU

$x - e^{x/5} = 0;\ \boxed{x = e^{x/5}}$

$x = 0 \longrightarrow f(0) = 0 - 1 = -1 < 0$

$x = 1 \longrightarrow f(1) = 1 - e^{1/5} = -0{,}22 < 0 \ (-)$

$x = 2 \longrightarrow f(2) = 2 - e^{2/5} = 0{,}508 > 0 \ (+)$ $\quad \boxed{P_1 \in [1,2]}$

Jakinda errorik handiena 10 baino handiagoa dela:

$x = 10 \longrightarrow f(10) = 10 - e^{2} > 0$

$x = 11 \longrightarrow f(11) = 11 - e^{11/5} > 0$

$x = 12 \longrightarrow f(12) = 12 - e^{12/5} > 0 \ (+)$

$x = 13 \longrightarrow f(13) = 13 - e^{13/5} < 0 \ (-)$ $\quad \boxed{P_2 \in [12,13]}$

B) REGULA FALSI bi aldiz erabiliz tarteak murriztu.

① $P_1 \in [1,2]$

• $x_1 = \dfrac{a_1 f(b_1) - b_1 f(a_1)}{f(b_1) - f(a_1)} = \dfrac{1\, f(2) - 2 f(1)}{f(2) - f(1)} = \dfrac{0{,}508175 + 0{,}442806}{0{,}508175 + 0{,}221403};$

$\boxed{x_1 = 1{,}30347}$

$\hookrightarrow f(x_1) = f(1{,}30347) > 0 \longrightarrow P_1 \in [\overset{(-)}{1};\ \overset{(+)}{1{,}30347}]$

• $x_2 = \dfrac{a_2 f(b_2) - b_2 f(a_2)}{f(b_2) - f(a_2)} = \dfrac{1 \cdot f(1{,}30347) - 1{,}30347\, f(1)}{f(1{,}30347) - f(1)} = \dfrac{0{,}00563953 + 0{,}282592}{0{,}00563953 + 0{,}221403};$

$\boxed{x_2 = 1{,}29593}$

② $P_2 \in [12,13]$

• $x_1 = \dfrac{12 f(13) - 13 f(12)}{f(13) - f(12)} = \dfrac{-5{,}56486 - 12{,}6987}{-1{,}44056};$ $\boxed{x_1 = 12{,}6781}$

$\rightarrow f(x_1) = f(12{,}6781) > 0 \longrightarrow P_2 \in [12{,}6781;\ 13]$

• $x_2 = \dfrac{12{,}6781\, f(13) - 13 f(12{,}6781)}{f(13) - f(12{,}6781)} = \dfrac{-5{,}87932 - 0{,}699979}{-0{,}517583};$ $\boxed{x_2 = 12{,}7116}$

c) PUNTU FINKOKO METODOA

g BILATU x ASKATUZ

$f(x) = x - e^{x/5}$

① $x = \boxed{e^{x/5} = g_1(x)}$

↓ EGIAZTATZEKO

$g_1'(x) = \frac{1}{5} e^{x/5}$

$\hookrightarrow g_1'(P_1) \approx g_1'(1,29593) = |0,259175| < 1 \longrightarrow$ BALIO DU $|g_1'(P_1)| < 1$ BETE

$$\boxed{X_3 = g_1(X_2) = 1,29587}$$

② $x = e^{x/5} = g_2(x)$

↓ EGIAZTATZEKO

$g_1'(x) = \frac{1}{5} e^{x/5}$

$\hookrightarrow g_1'(P_2) \approx g_2'(12,7116) = |2,541821| > 1 \rightarrow$ EZ DU BALIO $|g_1'(x)| > 1$

BESTE g BAT TOPATU

$x - e^{x/5} = 0$; $x = e^{x/5}$; $\ln x = \ln e^{x/5}$; $\ln x = \frac{x}{5}$; $x = \boxed{5 \ln x = g_2}$

EGIAZTATZEKO $\longrightarrow$ $g_2'(x) = 5 \frac{1}{x} = \frac{5}{x}$

$\hookrightarrow g_2'(P_2) \approx g_2'(12,7116) = |0,393341| < 1 \rightarrow$ BALIO DU $|g_2'(x)| < 1$ BETE

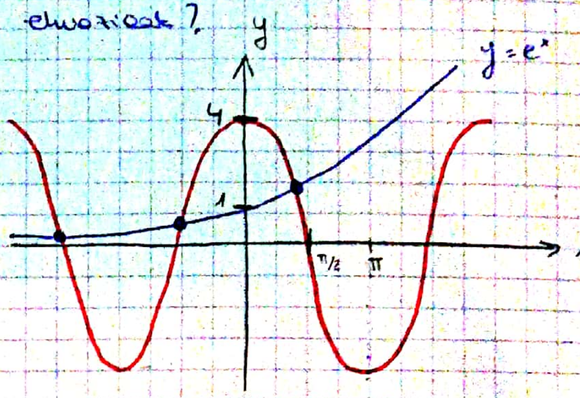
$$\boxed{X_3 = g_2(X_2) = 12,7126}$$

$X_2 \rightarrow$ REGULA FALSI EGINEZ LORTU DUGU

15. Zenbat soluzio $4\cos x - e^x = 0$ ekuazioak?

$4\cos x - e^x = 0$; $\boxed{4\cos x = e^x}$

3 SOLUZIO



• **NEWTON - RAPHSON** $\varepsilon = 10^{-4}$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

FORMULA APLIKATUZ

$$f(x) = 4\cos x - e^x \xrightarrow{\text{Deribatu}} f'(x) = -4\sin x - e^x$$

• $x_0 = 1$ (ENUNTZIATUTIK)

• $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{4\cos(1) - e^1}{-4\sin(1) - e^1} = 1{,}45$ → $e = |1{,}45 - 1| = 0{,}45 > \varepsilon$

• $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1{,}45 - \frac{4\cos(1{,}45) - e^{1{,}45}}{-4\sin(1{,}45) - e^{1{,}45}} = 0{,}99$ → $e = |0{,}99 - 1{,}45| = 0{,}46 > \varepsilon$

• $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0{,}99 - \frac{4\cos(0{,}99) - e^{0{,}99}}{-4\sin(0{,}99) - e^{0{,}99}} = 0{,}908$ → $e = |0{,}908 - 0{,}99| = 0{,}082 > \varepsilon$

• $x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0{,}908 - \frac{4\cos(0{,}908) - e^{0{,}908}}{-4\sin(0{,}908) - e^{0{,}908}} = 0{,}90472$ → $e = |0{,}904 - 0{,}908| = 0{,}004 > \varepsilon$

• $x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 0{,}904 - \frac{4\cos(0{,}904) - e^{0{,}904}}{-4\sin(0{,}904) - e^{0{,}904}} = 0{,}90478$ → $e = |0{,}90478 - 0{,}90472| = 0{,}00006 < \varepsilon$

ETEN!

• **EBAKITZAILEAREN METODOA**

$x_0 = \frac{\pi}{4}$ ETA $x_1 = \frac{\pi}{2}$ IZANIK

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)\cdot(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}}$$

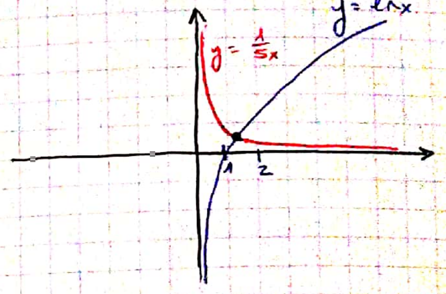
$$x_2 = x_1 - \frac{f(x_1)\cdot(x_1 - x_0)}{f(x_1) - f(x_0)} = \frac{\pi}{2} - \frac{f(\frac{\pi}{2})\cdot\frac{\pi}{4}}{f(\frac{\pi}{2}) - f(\frac{\pi}{4})} = \frac{\pi}{2} - \frac{-4{,}81047\cdot\frac{\pi}{4}}{(-4{,}81047) - 0{,}635147} = 0{,}877003$$

(13) $f(x) = 5x \ln x - 1$

$\begin{cases} t = 6 \text{ digitu esanguratsu} \\ \% 0{,}1 \text{ zehaztasunez} \rightarrow \varepsilon = 0{,}1 \end{cases}$

EKUAZIOA ERALDATU GRAFIKATZEKO

$5x \ln x - 1 = 0; \; 5x \ln x = 1; \; \ln x = \frac{1}{5x}$



① BISEKZIO METODOA

$\exists p \in [1,2]$

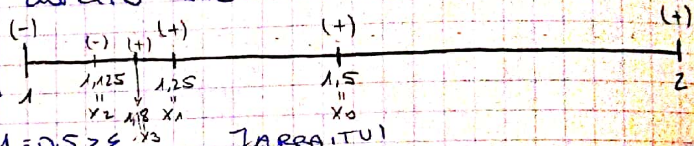
$f(1) = 5 \ln(1) - 1 = -1 < 0$

$f(2) = 10 \ln 2 - 1 > 0$

f jarraitua

→ BISEKZIO METODOA APLIKA DAITEKE

0,1 balio luzera txikiagoa duen tarte bat aurkitu arte



(0) $\begin{cases} x_0 = \frac{a_0 + b_0}{2} = 1{,}5 \\ f(x_0) = 2{,}0409... > 0 \end{cases}$ → $\exists p \in [1; 1{,}5]$ luz $= 1{,}5 - 1 = 0{,}5 > \varepsilon$ JARRAITU!

(1) $\begin{cases} x_1 = \frac{a_1 + b_1}{2} = 1{,}25 \\ f(x_1) = 0{,}3946... > 0 \end{cases}$ → $\exists p \in [1; 1{,}25]$ luz $= 1{,}25 - 1 = 0{,}25 > \varepsilon$ JARRAITU!

(2) $\begin{cases} x_2 = \frac{a_2 + b_2}{2} = 1{,}125 \\ f(x_2) = -0{,}3374... < 0 \end{cases}$ → $\exists p \in [1{,}125; 1{,}25]$ luz $= 1{,}25 - 1{,}125 = 0{,}125 > \varepsilon$ JARRAITU!

(3) $\begin{cases} x_3 = \frac{a_3 + b_3}{2} = 1{,}1875 \\ f(x_3) = 0{,}0203... > 0 \end{cases}$ → $\exists p \in [1{,}125; 1{,}1875]$ luz $= 1{,}1875 - 1{,}125 = 0{,}0625 < \varepsilon$ $\boxed{\text{ETEN !!!}}$

→ $p \approx x_4 = 1{,}15625$

② PUNTU FINKOKO METODOA

g BILATU x ASKATUZ

(1) $5x \ln x - 1 = 0; \; 5x \ln x = 1; \; x = \boxed{\frac{1}{5 \ln x} = g_1(x)}$

EGIAZTATU

$g_1'(x) = \frac{1}{5} \frac{-1/x}{\ln^2(x)} = -\frac{1}{5x \ln^2(x)}$

EZ DU BALIO

→ $g_1'(p) \approx g_1'(1{,}15625) = -\frac{1}{(5 \cdot 1{,}15625) \ln^2(1{,}15625)} = -8{,}206... \rightarrow |g_1'(x)| > 1$

BESTE g BAT BILATU

(2) $5x \ln x - 1 = 0; \; 5x \ln x = 1; \; \ln x = \frac{1}{5x}; \; e^{\ln x} = e^{1/5x}; \; x = \boxed{e^{1/5x} = g_2(x)}$

EGIAZTATU

$g_2'(x) = \frac{1}{5} e^{1/5x}$

BALIO DU

→ $g_2'(p) \approx g_2'(1{,}15625) = \frac{1}{5} e^{1/5 \cdot 1{,}15625} = 0{,}237... \rightarrow |g_2'(x)| < 1$

$x_0 = 1,15625$

$x_1 = g(x_0) = 1,18883$

$x_2 = g(x_1) = 1,18321$

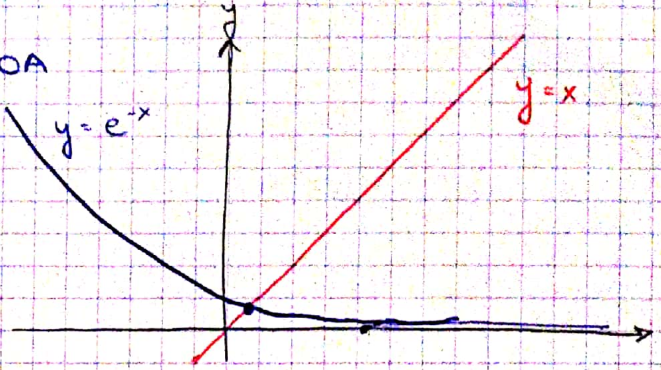
$e = \frac{|1,18883 - 1,15625|}{1,18883} = \%2,7 > \%0,1$ JARRAITU

$e = \frac{|1,18321 - 1,18883|}{1,18321} = \%0,47 > \%0,1$ JARRAITU

$x_3 = g(x_2) = 1,18416$ $\}$ $e = \%0,08 < \%0,1$ $\boxed{\text{ETEN!}}$

$$\boxed{p \approx x_3 = 1,18416}$$

10) PUNTU FINKOKO METODOA



$g(x) = e^{-x} = x$

$t = 6$ digitu esanguratsu

NON $p \in [0,5625; 0,625]$

- $x_0 = 0$ (HASTEKO)
- $x_1 = g(x_0) = 1$
- $x_2 = g(x_1) = \frac{1}{e} = 0,367879$
- $x_3 = g(x_2) = 0,692201$
- $x_4 = g(x_3) = 0,500473$
- $x_5 = g(x_4) = 0,606243$

ORAIN, metodoak konbergitzen duen egiaztatuko dugu, HAU DA, $|g'(x)| < 1$? NON $g'(x) = -e^{-x}$

$g'(p) \approx g'(0,606243) = -e^{-0,606243} = -0,545395 \rightarrow$ KONBERGITZEN DU $|g'(x)| < 1$ BAITA

$$e = \frac{|0,606243 - 0,500473|}{0,606243} = \%0,17$$

ZEINEK MEREZI DU GEHIAGO, BISEKZIO METODOAK EDO PUNTU FINKOKO METODOAK?

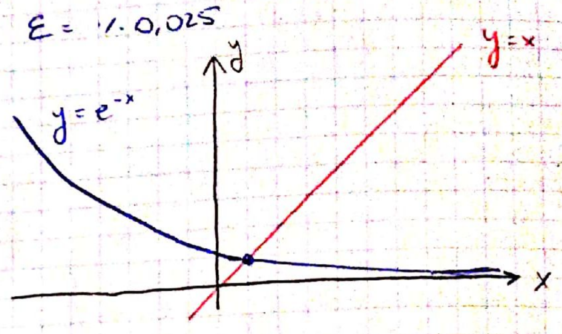
(11) NEWTON - RAPHSON

$t = 6$ digitu esanguratsu

$\varepsilon = \%\, 0{,}025$

$p \in [0{,}5625 ; 0{,}625]$

$g(x) = e^{-x} = x$; $\boxed{f(x) = e^{-x} - x}$



p hurbildua lortzeko:

$$p \approx \frac{0{,}5625 + 0{,}625}{2} = 0{,}59375$$

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}} \quad \text{NON} \quad \boxed{f'(x) = -e^{-x} - 1}$$

$x_0 = 0{,}59375$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0{,}59375 - \frac{-e^{-0{,}59375} - 1}{-1{,}55225} = 0{,}567016$$

$\Big\} \; e = \left| \frac{x_1 - x_0}{x_1} \right| = \%\, 4{,}7 > \varepsilon$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0{,}567146$$

$\Big\} \; e = \left| \frac{x_2 - x_1}{x_2} \right| = \%\, 0{,}022 < \varepsilon$ $\boxed{\text{ETEN!}}$

HORTAZ $\longrightarrow$ $\boxed{p \approx x_2 = 0{,}567146}$

(12) $f(x) = x^3 + 2x^2 + 10x - 20$

$t = 6$ digitu esanguratsu

$x^3 + 2x^2 + 10x - 20 = 0$; $x^3 = -2x^2 - 10x + 20$ ⊛

⊛ PARABOLAREN ERPINA ATERATZEKO

$$\frac{d}{dx}(-2x^2 - 10x + 20) = -4x - 10 = 0 ; \boxed{x = -2{,}5}$$ PARABOLAREN ERPINA

$\exists p [0, 2]$

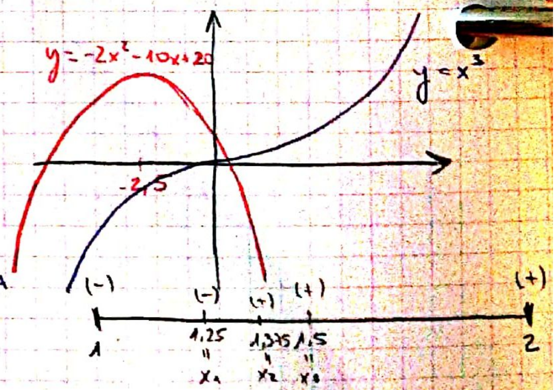
$f(0) = -20 < 0$

$f(1) = -7 < 0$

$f(2) = 8 + 8 + 20 - 20 > 0$

f jarraitua

$\rightarrow$ BISEKZIO METODOA APLIKA DAITEKE



(0) $\begin{cases} x_0 = \frac{a_0 + b_0}{2} = 1{,}5 \\ f(x_0) = 2{,}875 > 0 \end{cases}$ $\longrightarrow$ $\exists p \in [1 ; 1{,}5]$

(1) $\begin{cases} x_1 = \frac{a_1 + b_1}{2} = 1{,}25 \\ f(x_1) = -2{,}421 < 0 \end{cases}$ $\longrightarrow$ $\exists p \in [1{,}25 ; 1{,}5]$

(2) $\begin{cases} x_2 = \frac{a_2 + b_2}{2} = 1{,}375 \\ f(x_2) = 0{,}13085 > 0 \end{cases}$ $\longrightarrow$ $\exists p \in [1{,}25 ; 1{,}375]$

$\boxed{\text{REGULA FALSI APLIKATU}}$ $$x_1 = \frac{a_1 \cdot f(b_1) - b_1 \cdot f(a_1)}{f(b_1) - f(a_1)}$$

$$x_3 = \frac{(1{,}25 \cdot 0{,}130859) - (1{,}375 \cdot (-2{,}421871))}{0{,}130859 - (-2{,}421871)} = \boxed{1{,}36859}$$

$\boxed{\text{HORNER APLIKATU}}$

	1	2	10	-20
x_3		1,36859	4,61022	19,9953
	1	3,36859	14,6102	-0,004462638 $= f(x_3)$
x_3		1,36859	6,48326	
	1	4,73718	21,0934 $= f'(x_3)$	

$$\boxed{x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1{,}36881}$$

(2) $p(x) = x^3 - 3x^2 + 3x - 1$ $t = 3$ digitu NON $p(x) = a_0 x^3 + a_1 x^2 + a_2 x + a_3$

- $x = 2{,}19$ puntuan zuzenean

  $p(2{,}19) = (2{,}19)^3 - 3(2{,}19)^2 + 3(2{,}19) - 1 = 1{,}68$

- HÖRNER $x = 2{,}19$ izanik

  $a_0 = 1$ $b_0 = a_0 = 1$

  $a_1 = -3$ $b_1 = a_1 + b_0 x = -3 + (1 \cdot 2{,}19) = -0{,}81$

  $a_2 = 3$ $b_2 = a_2 + b_1 x = 3 + (-0{,}81 \cdot 2{,}19) = 1{,}226$

  $a_3 = -1$ $b_3 = a_3 + b_2 x = (-1) + (1{,}226 \cdot 2{,}19) = 1{,}69$

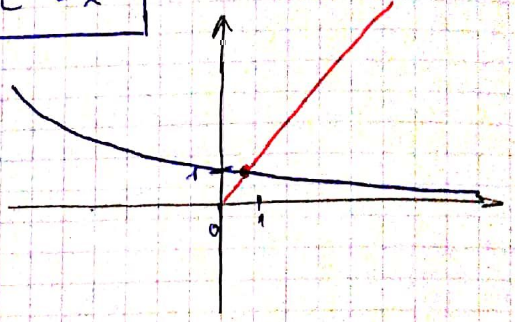
- ZEHATZA

  $p(2{,}19) = 1{,}685159$

⑧ REGULA FALSI

$\begin{cases} t=4 \text{ zifra esanguratsuz} \\ \%1 \text{ zehaztasuna.} \end{cases}$

$g(x) = x = e^{-x};$ $\longrightarrow$ $\boxed{f(x) = e^{-x} - x}$



MUTUR OSOAK dituen tarte bat

↳ BOLZANO $(0,1)$

$f(0) = e^{0} - 0 > 0 \quad (+)$

$f(1) = e^{-1} - 1 < 0 \quad (-)$

BOLZANO BETE ✓ $f(a) \cdot f(b) < 0$ BAITA

$$\boxed{X_n = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}}$$

(1) $X_1 = \dfrac{0\, f(1) - 1\, f(0)}{f(1) - f(0)} = 0{,}6127$

$f(X_1) = -0{,}07083 < 0 \quad (-)$ $\begin{cases} a_1 = 0 \\ b_1 = 0{,}6127 \end{cases}$

(2) $X_2 = \dfrac{0\, f(0{,}6127) - 0{,}6127\, f(0)}{f(0{,}6127) - f(0)} = 0{,}5722$

$f(X_2) = -0{,}007917 < 0 \quad (-)$ $\begin{cases} a_2 = 0 \\ b_2 = 0{,}5722 \end{cases}$

(3) $X_3 = \dfrac{0\, f(0{,}5722) - 0{,}5722\, f(0)}{f(0{,}5722) - f(0)} = 0{,}5671$

$f(X_3) = 0{,}00006784 > 0 \quad (+)$ $\begin{cases} a_3 = 0{,}5671 \\ b_3 = 0{,}5722 \end{cases}$

$e = \dfrac{|0{,}5671 - 0{,}5722|}{0{,}5671} \cdot 100 = \% 0{,}89 \longrightarrow$ ETEN!!!

↓

$e = \dfrac{|X_3 - X_2|}{X_3}$

$$\boxed{p \approx 0{,}5671}$$

9 AURREKO ARIKETA EBAKITZAILEAREN METODOA ERABILIZ

EBAKITZAILEAREN METODOA $\longrightarrow$ $\boxed{X_{n+1} = X_n - \dfrac{f(X_n)\cdot(X_n - X_{n-1})}{f(X_n) - f(X_{n-1})}}$ (SECANTE)

$a_0 = 0$ ETA $b_0 = 1$ $\oplus$ $\boxed{f(x) = e^{-x} - x}$ $X_0 = 1$

- $X_1 = b_0 - \dfrac{f(b_0)\,(b_0 - a_0)}{f(b_0) - f(a_0)} = 0{,}6127$

- $X_2 = X_1 - \dfrac{f(X_1)\cdot(X_1 - X_0)}{f(X_1) - f(X_0)} = 0{,}6127 - \dfrac{-0{,}07083\cdot(0{,}6127-1)}{-0{,}07083 + 0{,}6321} = 0{,}5638$

$e = \dfrac{|X_2 - X_1|}{X_2} = \%\,8{,}67 > \%\,1$ $\boxed{\text{JARRAITU!}}$

- $X_3 = 0{,}5638 - \dfrac{0{,}005243\,(0{,}5638 - 0{,}6127)}{0{,}005243 + 0{,}07083} = 0{,}5672$

$e = \dfrac{|X_3 - X_2|}{X_3} = \%\,0{,}599 < \%\,1$ $\boxed{\text{ETEN!!!}}$ $\boxed{p \approx 0{,}5672}$

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