1. GAIA - TXOSTENEKO ARIKETAK

POLAR → BINOMIKO

1.1) $z = 5 \cdot e^{7\pi i} = 5 \cdot e^{\pi i} = 5_\pi = \boxed{-5}$

1.2) $z = \frac{1}{3} e^{-\frac{3\pi}{2} i} = \begin{cases} \rho = 1/3 \\ \theta = -3\pi/2 \end{cases}$

$z = \rho(\cos\theta + i\sin\theta) = \frac{1}{3}\left(\cancel{\cos\left(-\frac{3\pi}{2}\right)}^{0} + i\underbrace{\sin\left(-\frac{3\pi}{2}\right)}_{1}\right) = \boxed{\frac{1}{3} i}$

1.3) $z = e^{-\pi/6 i} = \begin{cases} \rho = 1 \\ \theta = -\pi/6 \end{cases}$

$z = \rho(\cos\theta + i\sin\theta) = 1\left(\cos(-\pi/6) + i\sin(-\pi/6)\right) = \boxed{\frac{\sqrt{3}}{2} - \frac{1}{2} i}$

1.5) $z = e^{7\pi/3 i} = \begin{cases} \rho = 1 \\ \theta = 7\pi/3 \end{cases}$

$z = \cos(7\pi/3) + i\sin(7\pi/3) = \boxed{\frac{1}{2} + \frac{\sqrt{3}}{2} i}$

1.6) $z = \sqrt{2}_\pi = \boxed{-\sqrt{2}}$

1.7) $z = \sqrt{2}_{\pi/4} = \begin{cases} \rho = \sqrt{2} \\ \theta = \pi/4 \end{cases}$

$z = \sqrt{2}\left(\cos(\pi/4) + i\sin(\pi/4)\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \boxed{1 + i}$

1.8) $z = 5_{-7\pi/2} = \begin{cases} \rho = 5 \\ \theta = -7\pi/2 \end{cases}$

$z = 5\left(\overset{0}{\cos(-7\pi/2)} + i\underbrace{\sin(-7\pi/2)}_{1}\right) = \boxed{5i}$

1.9) $z = \underset{\substack{\text{Alde} \\ \text{erreal} \\ \text{negatiboa}}}{1_\pi} \cdot \underset{\substack{\text{Alde} \\ \text{irudikari} \\ \text{positiboa}}}{2_{\pi/2}} = (-1) \cdot 2i = \boxed{-2i}$

BINOMIKO → POLAR

2.1) $z = 5i \longrightarrow$ Ardatz irudikariaren tarte positiboan.

$z = 5_{\pi/2} = \boxed{5 e^{i\pi/2}}$

2.2) $z = -(1 + \sqrt{3} i)$ $\begin{cases} a = -1 \\ b = -\sqrt{3} \end{cases}$ NON $a + bi$ BINOMIKOA (3. KOAD)

Modulua $\leftarrow$ $\begin{cases} \rho = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2 \text{ (Modulua denez } \oplus) \\ \alpha = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{-\sqrt{3}}{-1}\right) = \pi/3 \text{ (3. koadrantean egongo dela)} \end{cases}$

HORTAZ $\longrightarrow$ $\boxed{z = 2_{2\pi/3} = 2 e^{-i2\pi/3}}$ $\theta = \alpha - \pi = \pi/3 - \pi = -2\pi/3$

2.3) $z = -1 + i$ $\begin{cases} a = -1 \\ b = 1 \end{cases}$ > 2. KOADRANTEA

$\begin{cases} \rho = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + 1^2} = \sqrt{2} \\ \alpha = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{1}{-1}\right) = \pi/4 \\ \hookrightarrow \theta = \pi - \alpha = \pi - \pi/4 = 3\pi/4 \end{cases}$ $\boxed{z = \sqrt{2}_{3\pi/4} = \sqrt{2}\, e^{i\,3\pi/4}}$

2.4) $z = -1 - i \begin{cases} a = -1 \\ b = -1 \end{cases}$ > 3. KOADRANTEA $\boxed{\theta = \alpha - \pi}$

$$\begin{cases} \rho = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2} \\ \alpha = \arctan\left(\frac{-1}{-1}\right) = \pi/4 \end{cases}$$

$\hookrightarrow \theta = \alpha - \pi = \pi/4 - \pi = -3\pi/4$

$$\boxed{z = \sqrt{2}_{-3\pi/4} = \sqrt{2}\, e^{-i\,3\pi/4}}$$

2.5) $z = (-\sqrt{2} + \sqrt{2}\, i)^4 \begin{cases} a = -\sqrt{2} \\ b = \sqrt{2} \end{cases}$ > 1. KOADRANTEA

$$\begin{cases} \rho = \sqrt{(-\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{2+2} = \sqrt{4} = 2 \\ \theta = \arctan\left(\frac{\sqrt{2}}{-\sqrt{2}}\right) = +\pi/4 \text{ (1. koadrantea)} \end{cases}$$

$$z = (2\, e^{i\,\pi/4})^4 = \boxed{16\, e^{i\pi/4}}$$

2.6) $z = (-2 - 2i)^2 \begin{cases} a = -2 \\ b = -2 \end{cases}$ > 3. KOADRANTEA $\boxed{\theta = \alpha - \pi}$

$$\begin{cases} \rho = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} \\ \theta = \arctan\left(\frac{-2}{-2}\right) = \pi/4 \end{cases}$$

$$\boxed{z = (\sqrt{8} \cdot e^{i\pi/4})^2 = 8 \cdot e^{i\pi/2}}$$

2.8) $z = i\,(1+i)\, e^{\pi/6\, i}$

① $i = e^{\pi/2\, i}$

② $1 + i = \sqrt{2}\, e^{\pi/4\, i}$

$$z = e^{\pi/2\, i} \cdot \sqrt{2}\, e^{\pi/4\, i} \cdot e^{\pi/6\, i}$$

$$\frac{\pi}{2} + \frac{\pi}{4} + \frac{\pi}{6} = \frac{12\pi}{24} + \frac{6\pi}{24} + \frac{4\pi}{24} = \frac{22\pi}{24} = \frac{11\pi}{12}$$

HORTAZ $\hookrightarrow \boxed{z = \sqrt{2}\, e^{11\pi/12\, i}}$

NORMALEAN, hobe da dena esponentzialera pasatzea berretzaileak elkartu ahal izateko.

2.9) $z = (\sqrt{3} + i) \cdot 2\sqrt{2}\, e^{-\pi/4\, i}$

⊗ $\begin{cases} a = \sqrt{3} \\ b = 1 \end{cases} \longrightarrow \begin{cases} \rho = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2 \\ \theta = \arctan\left(\frac{1}{\sqrt{3}}\right) = \pi/6 \end{cases}$

$$z = 2e^{\pi/6\, i}\; 2\sqrt{2}\; e^{-\pi/4\, i}$$

$\hookrightarrow \frac{\pi}{6} - \frac{\pi}{4} = \frac{4\pi}{24} - \frac{6\pi}{24} = -\frac{\pi}{12}$

HORTAZ $\hookrightarrow \boxed{z = 4\sqrt{2}\, e^{-\pi/12\, i}}$

2.7) $z = \dfrac{1 + i\sqrt{3} \;①}{\sqrt{3} + i \;②}$

① $\begin{cases} a = 1 \\ b = \sqrt{3} \end{cases} \longrightarrow \begin{cases} \rho = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2 \\ \theta = \arctan\left(\frac{\sqrt{3}}{1}\right) = \pi/3 \end{cases} \; 2e^{\pi/3\, i}$

② $\begin{cases} a = \sqrt{3} \\ b = 1 \end{cases} \longrightarrow \begin{cases} \rho = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2 \\ \theta = \arctan\left(\frac{1}{\sqrt{3}}\right) = \pi/6 \end{cases} \; 2e^{\pi/6\, i}$

⊗ $\frac{\pi}{3} - \frac{\pi}{6} = \frac{2\pi}{6} - \frac{\pi}{6} = \frac{\pi}{6}$

HORTAZ $\hookrightarrow z = \dfrac{2e^{\pi/3\, i}}{2e^{\pi/6\, i}} = e^{(\pi/3 - \pi/6)\, i} ⊗ \,,\; \boxed{z = e^{\pi/6\, i}}$

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4.4) $z^3 + z = 0 \longrightarrow z(z^2+1) = 0 \longrightarrow \boxed{z_1 = 0}$

$$z = \frac{-0 \pm \sqrt{0^2 - 4\cdot 1\cdot 1}}{2\cdot 1} = \frac{\pm\sqrt{-4}}{2} = \frac{\pm 2\sqrt{-1}}{2} = \pm i \begin{cases}\boxed{z_2 = i}\\ \boxed{z_3 = -i}\end{cases}$$

4.5) $z^3 - z = 0 \longrightarrow z(z^2-1) = 0 \longrightarrow \boxed{z_1 = 0}$

$$z = \frac{-0 \pm \sqrt{0 - 4\cdot 1\cdot (-1)}}{2\cdot 1} = \frac{\pm\sqrt{4}}{2} = \pm\frac{2}{2} = \pm 1 \begin{cases}\boxed{z_2 = 1}\\ \boxed{z_3 = -1}\end{cases}$$

3.1) $\sqrt[3]{27i} = (27i)^{1/3} = (27_{\pi/2})^{1/3}$ $\qquad 120° = 2\pi/3$ rad IZANIK

(1) $z_1 = (27)^{1/3} e^{i\theta_1} = 3e^{\pi/6 i} = 3_{\pi/6}$

$\hookrightarrow z_1 = 3(\cos(\pi/6) + i\sin(\pi/6)) = 3\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \boxed{\frac{3}{2}(\sqrt{3}+i)}$

(2) $z_2 = 3_{\pi/6 + 2\pi/3} = 3_{\pi/6 + 4\pi/6} = 3_{5\pi/6} = 3e^{5\pi/6 i}$

$\hookrightarrow z_2 = 3(\cos(5\pi/6) + i\sin(5\pi/6)) = 3\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \boxed{\frac{3}{2}(-\sqrt{3}+i)}$

(3) $z_3 = 3_{5\pi/6 + 2\pi/3} = 3_{5\pi/6 + 4\pi/6} = 3_{9\pi/6} = 3_{3\pi/2} = 3e^{3\pi/2 i}$

$\hookrightarrow z_3 = 3(\cos(3\pi/2) + i\sin(3\pi/2)) = 3(0 - i) = \boxed{-3i}$

9) $(\underset{①}{1+i\sqrt{3}})^n + (\underset{②}{1-i\sqrt{3}})^n = 2^{n+1}\cdot\cos(n\pi/3)$

① $1+i\sqrt{3} \begin{cases}a=1\\ b=\sqrt{3}\end{cases} \longrightarrow \begin{cases}\rho = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{4} = 2\\ \theta = \arctan(\sqrt{3}/1) = \pi/3\end{cases} 2e^{\pi/3 i}$

② $1-i\sqrt{3} \begin{cases}a=1\\ b=-\sqrt{3}\end{cases} \longrightarrow \begin{cases}\rho = \sqrt{1^2+(-\sqrt{3})^2} = \sqrt{4} = 2\\ \theta = \arctan(-\sqrt{3}/1) = -\pi/3\end{cases} 2e^{-\pi/3 i}$

$\hookrightarrow (2e^{\pi/3 i})^n + (2e^{-\pi/3 i})^n = 2^n e^{n\pi/3 i} + 2^n e^{-n\pi/3 i} =$

$= 2^n\left(e^{n\pi/3 i} + e^{-n\pi/3 i}\right) \overset{⊗}{=} 2^n\cdot 2\cos(n\pi/3) = \underline{2^{n+1}\cos(n\pi/3)}$

⊗ OHARRA!

$e^{ix} = \cos x + i\sin x$

$e^{-ix} = \cos x - i\sin x$

$e^{ix} + e^{-ix} = 2\cos x$

8) MOIVRE-ren FORMULA $\rightarrow (\cos\theta + i\sin\theta)^n = [\cos(n\theta) + i\sin(n\theta)]$ $n \in \mathbb{Z}$

$\rho = 1$ izanik ETA $z = \rho(\cos\theta + i\sin\theta)$

ORDUAN $\hookrightarrow z^n = (\cos\theta + i\sin\theta)^n = (1e^{\theta i})^n = e^{\theta n i} = 1_{n\theta}$

ALDI BEREAN, $1_{n\theta} = [\cos(n\theta) + i\sin(n\theta)]$

HORTAZ, $(\cos\theta + i\sin\theta)^n = [\cos(n\theta) + i\sin(n\theta)]$

6) UNITATEAREN (1) 6 ERROAK $\longrightarrow \pi/3$

$6\theta = 0 + 2k\pi,\ \theta = \frac{2k\pi}{6}$ $\quad \begin{cases} \theta_1 = 0 & k=0 \\ \theta_2 = \pi/3 & k=1 \\ \theta_3 = 2\pi/3 & k=2 \\ \theta_4 = \pi & k=3 \\ \theta_5 = 4\pi/3 & k=4 \\ \theta_6 = 5\pi/3 & k=5 \end{cases}$

① $z_1 = 1^{1/6} e^{i0} = 1_0$

② $z_2 = 1_{0+\pi/3} = 1_{\pi/3}$

↳ $z_2 = 1\,[\cos(\pi/3) + i\sin(\pi/3)] = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

③ $z_3 = 1_{\pi/3+\pi/3} = 1_{2\pi/3}$

↳ $z_3 = 1\,[\cos(2\pi/3) + i\sin(2\pi/3)] = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

④ $z_4 = 1_{2\pi/3+\pi/3} = 1_{3\pi/3} = 1_{\pi} = -1$

⑤ $z_5 = 1_{\pi+\pi/3} = 1_{3\pi/3+\pi/3} = 1_{4\pi/3}$

↳ $z_5 = 1\,[\cos(4\pi/3) + i\sin(4\pi/3)] = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

⑥ $z_6 = 1_{4\pi/3+\pi/3} = 1_{5\pi/3}$

↳ $z_6 = 1\,[\cos(5\pi/3) + i\sin(5\pi/3)] = \frac{1}{2} - i\frac{\sqrt{3}}{2}$

4.3) $(1+i)z^3 - 2i = 0$ 3 SOLUZIO $\longrightarrow$ $2\pi/3$ - 10

$z^3 = \frac{2i}{1+i} ⊛ \frac{(1-i)}{(1-i)} = \frac{2i+2}{2} = \frac{2(i+1)}{2} = i+1$

⊛ Konjokatua erabili i izendatzailetik kentzeko

HORTAZ

↳ $z^3 = 1+i$; $z = \sqrt[3]{1+i}$ NON $\begin{cases} \rho = \sqrt{1+1} = \sqrt{2} \\ \theta = \arctan(1/1) = \pi/4 \end{cases}$

$\frac{\text{ZENBAKITZAILE}}{\text{IZENDATZAILE}}$

$z = (\sqrt{2}_{\pi/4})^{1/3} = (\sqrt{2})^{1/3}_{\pi/12} = \sqrt[6]{2}_{\pi/12}$

$z_1 = \sqrt[6]{2}_{\pi/12}$

$z_2 = \sqrt[6]{2}_{\pi/12+2\pi/3} = \sqrt[6]{2}_{\pi/12+8\pi/12} = \sqrt[6]{2}_{9\pi/12}$

$z_3 = \sqrt[6]{2}_{9\pi/12+2\pi/3} = \sqrt[6]{2}_{9\pi/12+8\pi/12} = \sqrt[6]{2}_{17\pi/12}$

EDO

↳ θ LORTZEKO $\longrightarrow$ $\underbrace{3}_{\text{3 soluzio}}\theta = \underbrace{\frac{\pi}{4}}_{z\text{-ren angelua}} + 2k\pi$; $\theta = \frac{\pi + 8k\pi}{12}$ $\quad \begin{cases} \theta_1 = \pi/12 & k=0 \\ \theta_2 = 9\pi/12 & k=1 \\ \theta_3 = 17\pi/12 & k=2 \end{cases}$

10) $z^3 - 17iz^2 + (4i - 91)z + (171i + 36) = 0$

$z^3 = 27$, $z = \sqrt[3]{27}$, $z = 3$ $\xrightarrow[\text{IRUDIKARIA}]{\text{HIRUKOITZA}}$ $\boxed{z_1 = 9i}$

$$\begin{array}{c|cccc} & 1 & -17i & 4i-91 & 171i+36 \\ 9i & & 9i & 72 & -171i-36 \\ \hline & 1 & -8i & 4i-19 & \boxed{0} \end{array} \longrightarrow z^2 - 8iz + 4i - 19 = 0$$

$z = \frac{-(-8i) \pm \sqrt{(-8i)^2 - 4\cdot 1\cdot(4i-19)}}{2\cdot 1} = \frac{8i \pm \sqrt{-64-16i+76}}{2} = \frac{8i \pm \sqrt{12-16i}}{2} =$

$= \frac{8i \pm 2\sqrt{3-4i}}{2} = 4i \pm \sqrt{3-4i}$ $\longrightarrow$ $\boxed{z_2 = 4i + (2-i)}$

$\longrightarrow$ $\boxed{z_3 = 4i - (2-i)}$

⊛ 5. ARIKETA APLIKATUZ

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5) $\sqrt{3-4i}$ BINOMIKOA → Zenbaki konplexu guztiak

$$\sqrt{3-4i} = \{w_1, \underset{-w_1}{w_2}\} = \{w \mid w^2 = 3-4i\} = \{a+bi \mid (a+bi)^2 = 3-4i\}$$

$$(a+ib)^2 = a^2 - b^2 + 2abi = 3-4i \quad \boxed{i^2=-1} \qquad \begin{cases} a^2-b^2=3 \\ 2ab=-4 \end{cases}$$

$$a = \frac{-4}{2b};\ \boxed{a=-\frac{2}{b}} \xrightarrow{\text{HORTAZ}} \left(-\frac{2}{b}\right)^2 - b^2 = 3;\ \frac{4}{b^2} - b^2 = 3;\ 4-b^4 = 3b^2;$$

$$b^4 + 3b^2 - 4 = 0;\ (b^2)^2 + 3(b^2) - 4 = 0$$

$$b^2 = \frac{-3 \pm \sqrt{9+16}}{2} = \frac{-3\pm5}{2} \begin{cases} b^2 = 1;\ \boxed{b=\pm1} \checkmark \\ \cancel{-4} \text{ EZ DA POSIBLEA } \cancel{b^2=-4} \end{cases}$$

SOLUZIOAK

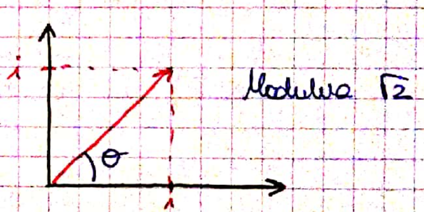
$$b=1 \longrightarrow a=-2 \longrightarrow \boxed{w_1 = -2+i}$$
$$b=-1 \longrightarrow a=2 \longrightarrow \boxed{w_2 = 2-i}$$
$$\sqrt{3-4i} = \{-2+i,\ 2-i\}$$

7.A)

$$z = A = \left(\frac{1+i}{\sqrt2}\right)^{10} + \left(\frac{1-i}{\sqrt2}\right)^{10}$$ → BERE KONJOKATUA

$$z = w^{10} + (\bar w)^{10} = w^{10} + (\overline{w^{10}}) \in \mathbb{R}$$

- $1+i \rightarrow$ Exponentzialean $\rightarrow \sqrt2\, e^{\pi/4\, i}$
- $1-i \rightarrow$ Exponentzialean $\rightarrow \sqrt2\, e^{-\pi/4\, i}$ ← KONJOKATUA BAITA



HORTAZ

$$z = \left(\frac{\sqrt2\, e^{\pi/4\, i}}{\sqrt2}\right)^{10} + \left(\frac{\sqrt2\, e^{-\pi/4\, i}}{\sqrt2}\right)^{10} = (e^{i\pi/4})^{10} + (e^{-i\pi/4})^{10} = e^{i10\pi/4} + e^{-i10\pi/4} =$$

$$= e^{i5\pi/2} + e^{-i5\pi/2} = (\underbrace{e^{i\pi/2}}_{i})^5 + (\underbrace{e^{-i\pi/2}}_{-i})^5 = \underbrace{(i)^5 + (-i)^5}_{i + (-i) = 0} = 0$$

$\boxed{\text{EDO}} \longrightarrow \cos(5\pi/2) + i\sin(5\pi/2) + \cos(5\pi/2) - i\sin(5\pi/2) = 0$

7.B)

$$z = B = \frac{1-e^{i\pi/2}}{1+e^{i\pi/2}} = \underbrace{\frac{1-i}{1+i}}_{\text{Aurreko ariketatik}} = \frac{\sqrt2\, e^{-i\pi/4}}{\sqrt2\, e^{i\pi/4}} = e^{-i(\pi/4+\pi/2)} = e^{-i\pi/2} = -i$$

4.1) $z^4 + 16 = 0$

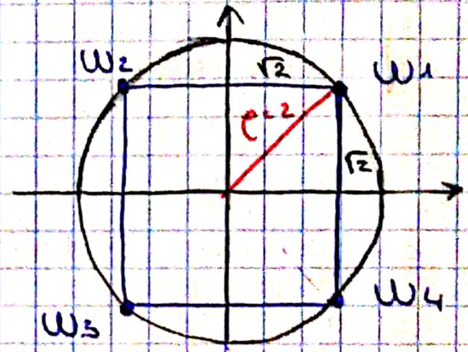
$z^4 = -16 = 16_{\pi} \longrightarrow (-16)^{1/4} = \{\omega_1, \omega_2, \omega_3, \omega_4\}$

$\omega_1 = 2_{\pi/4} = \sqrt{2}\,(1+i)$

$\omega_2 = -\omega_1 = \sqrt{2}\,(-1+i)$

$\omega_3 = \bar{\omega}_2 = \sqrt{2}\,(-1-i)$

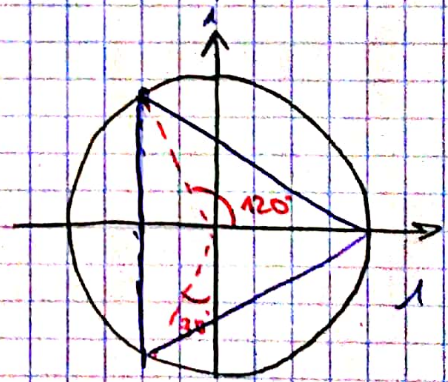
$\omega_4 = \bar{\omega}_1 = \sqrt{2}\,(1-i)$



4.2) $z^3 - 1 = 0$

RUFFINI

	1	0	0	-1
1		1	1	1
	1	1	1	0



HIRUKI ALDEKIDEA

1 erradioko zirkunferentzia

$z^3 - 1 = (z-1)\cdot(z^2+z+1) \rightarrow z = \dfrac{-1 \pm \sqrt{1^2-4}}{2} = -\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2} i$

HORTAZ $\longrightarrow$ $z^3 - 1 = (z-1)\cdot\left(z - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)\right)\cdot\left(z - \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right)\right)$

NON
$$\begin{cases} z_1 = 1 \\ z_2 = -1/2 + \sqrt{3}/2\, i = e^{2\pi/3\, i} \\ z_3 = -1/2 - \sqrt{3}/2\, i = e^{-2\pi/3\, i} \end{cases}$$

ZENBAKI KONPEXUAK - ARIKETA GEHIGARRIAK

$\boxed{1.}$ $z = \dfrac{e^{2\pi i \cdot n}}{2+2i}$ BINOMIKORA (NON n zenbaki osoa)

• $2+2i \begin{cases} a=2 \\ b=2 \end{cases} \longrightarrow \begin{cases} \rho = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2} \\ \theta = \arctan\left(\frac{2}{2}\right) = \pi/4 \end{cases} \Big\} \; 2\sqrt{2}\, e^{\pi/4\, i}$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

• $e^{2\pi i \cdot n} = 1^n = 1$

$\frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{4}$

↳ $z = \dfrac{1}{2\sqrt{2}\, e^{\pi/4\, i}} = \dfrac{\sqrt{2}}{4} e^{-\pi/4\, i} \rightarrow z = \rho(\cos\theta + i\sin\theta) =$

$= \dfrac{\sqrt{2}}{4} \cdot \left(\cos(-\pi/4) + i\sin(-\pi/4)\right) = \dfrac{\sqrt{2}}{4}\left(\dfrac{\sqrt{2}}{2} - i\dfrac{\sqrt{2}}{2}\right) = \boxed{\dfrac{1}{4} - i\dfrac{1}{4}}$

$\boxed{2.}$ $z = (1+i)^{36} - (\sqrt{3}-i)^{42}$ BINOMIKORA

• $1+i \begin{cases} a=1 \\ b=1 \end{cases} \longrightarrow \begin{cases} \rho = \sqrt{1^2+1^2} = \sqrt{2} \\ \theta = \arctan(1/1) = \pi/4 \end{cases} \Big\} \; \sqrt{2}\, e^{\pi/4\, i}$

• $\sqrt{3}-i \begin{cases} a=\sqrt{3} \\ b=-1 \end{cases} \longrightarrow \begin{cases} \rho = \sqrt{(\sqrt{3})^2+(-1)^2} = \sqrt{4} = 2 \\ \theta = \arctan\left(-\frac{1}{\sqrt{3}}\right) = -\pi/6 \end{cases} \Big\} \; 2e^{-\pi/6\, i}$

HORTAZ

↳ $z = (\sqrt{2}\, e^{\pi/4\, i})^{36} - (2e^{-\pi/6\, i})^{42} = 2^{18} e^{9\pi i} - 2^{42} \cdot e^{-7\pi i}$

• $z = 2^{18}\left(\cos(9\pi) + i\sin(9\pi)\right) = 2^{18} \cdot (-1) = -2^{18}$

• $z = 2^{42}\left(\cos(-7\pi) + i\sin(-7\pi)\right) = 2^{42}\;(-1) = -2^{42}$

↳ $z = -2^{18} + 2^{42}$

$\boxed{5.}$ A) i^{2012}

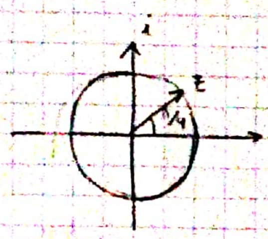
$2012 \,\underline{|\,4\,}$; 012 ; 0 ; 503 $\longrightarrow (i^4)^{503} = 1^{503} = 1$

B) $(1+i)^{2012} = \left((1+i)^4\right)^{503}$

• $i+1 = \sqrt{2}\, e^{\pi/4\, i} \longrightarrow \left((\sqrt{2}\, e^{\pi/4\, i})^4\right)^{503} = \left((\sqrt{2})^4 \cdot \overbrace{e^{\pi i}}^{-1}\right)^{503} = (-4)^{503} = -2^{1006}$

$\boxed{6}$ $z = (1+\sqrt{3}i)^{3/4}$

$\begin{cases} a=1 \\ b=\sqrt{3} \end{cases} \longrightarrow \begin{cases} \rho = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{4} = 2 \\ \theta = \arctan(\sqrt{3}/1) = \pi/3 \end{cases} \Big\} \; 2e^{\pi/3\, i}$



↳ $z = (2e^{\pi/3\, i})^{3/4} = 2^{3/4} e^{3\pi/12\, i} = 2^{3/4} e^{\pi/4\, i}$

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$\boxed{3}$ $Re\left[\frac{1}{1+e^{i\pi/3}}\right]$ NON $\begin{cases} Re(z) = a & \text{ERREALA} \\ Im(z) = b & \text{IRUDIKARIA} \end{cases}$

$$z = \frac{1}{1+\underset{①}{e^{i\pi/3}}} = \frac{e^{2\pi i}}{1+\frac{1}{2}+i\frac{\sqrt3}{2}} = \frac{e^{2\pi i}}{\underbrace{\frac{3}{2}+i\frac{\sqrt3}{2}}_{②}} = \frac{e^{2\pi i}}{\sqrt3 e^{\frac{\pi}{3}i}} = \frac{1}{\sqrt3} e^{-\frac{\pi}{6}i} \to ③$$

① BINOMIKORA $e^{i\pi/3} \to 1(\cos(\pi/3)+i\sin(\pi/3)) = \frac{1}{2} + i\frac{\sqrt3}{2}$

② POLARRERA $\begin{cases} a = 3/2 \\ b = \sqrt3/2 \end{cases} \longrightarrow \begin{cases} \rho = \sqrt{(3/2)^2+(\sqrt3/2)^2} = \sqrt3 \\ \theta = \arctan\left(\frac{\sqrt3/2}{3/2}\right) = \pi/6 \end{cases} \sqrt3\, e^{\frac{\pi}{6}i}$

③ BINOMIKORA $\frac{1}{\sqrt3} e^{-\frac{\pi}{6}i}$

$\hookrightarrow \frac{1}{\sqrt3}(\cos(-\pi/6)+i\sin(-\pi/6)) = \frac{1}{2} - \frac{\sqrt3}{6}i$

$\begin{cases} a = 1/2 \\ b = -\sqrt3/6 \end{cases}$ IZANIK, $Re\left[\frac{1}{1+e^{i\pi/3}}\right] = Re\left[\frac{1}{2} - \frac{\sqrt3}{6}i\right] = \frac{1}{2}$ ZATI ERREALA

$\boxed{4}$

A) $\frac{7e^{i\frac{3\pi}{4}}}{2e^{i\frac{2\pi}{3}}} = \frac{7}{2}e^{i\left(\frac{3\pi}{4}-\frac{2\pi}{3}\right)} = \frac{7}{2}e^{i\left(\frac{9\pi}{12}-\frac{8\pi}{12}\right)} = \frac{7}{2}e^{i\frac{\pi}{12}}$

B) $e^{i\pi}\left(\underset{①}{\frac{1}{2+i}}\right) = e^{i\pi}\left(\frac{2-i}{5}\right) = -\frac{2-i}{5} = -\frac{2}{5}+\frac{i}{5}$

① $\frac{1}{2+i}\cdot\frac{(2-i)}{(2-i)} = \frac{2-i}{4-2i+2i+1} = \frac{2-i}{5}$

$\boxed{7}$ $p(x)$ 4. MAILAKO ETA KOEFIZIENTE ERREALEKO POLINOMIO BAT.

A) $8i$ eta $5-3i$ polinomio horren 2 erro. BESTEAK?

BESTE BI ERROAK $\longrightarrow$ $\boxed{5+3i}$ ETA $\boxed{-8i}$

B) Demogun $p(x)$-ek erro erreal bat dueb. Zenbat erro irudikari izan ditzake? 3